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Principles of Physics Applied to Traffic Movements and Road Conditions

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SEVERAL recent articles and investigations[1] have shown an increasing realization of the need for a scientific approach to various aspects of road traffic, a field that seemed formerly to lie only at the periphery of physics. Important and fascinating as such studies are, there comes a point when the most intelligent observations of the procedure of *single vehicles* alone cannot furnish a complete explanation of what happens on the road. From there on we get a good deal further if, instead of focusing our attention on the individual vehicle, we begin to observe *traffic as a whole*—not merely as an accumulation of many independently moving cars, but as a sort of continuous stream, made up of many vehicles moving with continually changing density, yet not arbitrarily but in a definite relationship to one another and subject to common conditions. Indeed we know that almost never can the mechanical scope of a modern automobile be fully utilized, because the simultaneous actual or at least potential presence of other cars brings about limitations which force all vehicles to adopt typical patterns of behavior if they want to move at all. Perhaps the most outstanding condition produced by these limitations is that every vehicle requires a certain empty space around itself in order to proceed without interference with any other objects. This space moves with the vehicle as its core and is determined by the necessity for stopping clear of any obstruction. In other words, it must be large enough to allow the driver to perceive the obstruction early enough to react and to decelerate as much as necessary. We shall call this space the *influence space*, indicating that what comes within this space necessarily and immediately influences the procedure of the vehicle. Thus we can say with good reason that traffic is a stream consisting of influence spaces in motion.

Since such influence is generally exerted along the whole approach front and the major part of the sides of the vehicle, this influence space is required mainly in front and to some extent at either side. It has not yet been possible to make the necessary experiments to determine the exact shape of the influence space, which probably is somewhat like that shown in Fig. 1, but in any

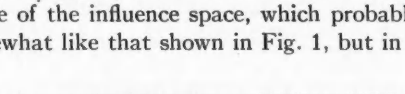

FIG. 1. Influence space, probable shape.

[1] For example: Chapman, Am. J. Phys. 10, 22 (1942); Fountain, Am. J. Phys. 10, 322 (1942).

case approaches a rectangular shape; the higher the speed, the less significant is the deviation from this shape. Therefore we assume at this stage of the studies as a workable approximation that the influence space is a rectangle (see Fig. 2), its width being equal to the average width of one traffic lane, whereas its length—the so-called *influence length*—is composed of the following three parts: (*i*) the perception way, or distance traveled during the perception (and reaction) time of the driver; (*ii*) the braking way, or minimum distance required to come to a stop after application of the brakes; (*iii*) the length of the car plus the required minimum distance from the obstruction when it has come to rest. Therefore, for motion at constant speed v the influence length l is given by

$$l = pv + \tfrac{1}{2}v^2/r + L, \tag{1}$$

where p is the perception plus reaction time (we have used the value $\frac{1}{2}$ sec.), r the maximum possible deceleration and L the length of the car plus the minimum distance between stationary cars. Thus the influence length depends on perception time, maximum possible deceleration and vehicle length as parameters. It increases rapidly with the speed.

As illustrated in Fig. 3, every vehicle obviously is forced to maneuver on the road in such a manner that its influence space does not interfere with those of others. However, the nature and purpose of highway traffic are such that its task of discharge must be performed not only *safely* but also as *economically* and as *speedily* as possible. Safety is assured if the influence spaces do not overlap, economy of traffic if there are no wide gaps between the influence spaces. The road is filled to capacity if the influence spaces just touch one another. The capacity c of a highway lane is equal to the number of influence spaces that can pass a given point of the lane per second; that is, capacity c equals the ratio v/l of speed to influence length. Substituting the value for the influence length from Eq. (1), we get

$$c = \frac{v}{pv + (\frac{1}{2}v^2/r) + L}.$$

This shows that the capacity of the same highway is different for different speeds. It is a maximum for a certain ***best speed*** v_0 which is found by differentiating the foregoing expression with respect to v and setting the result equal to zero; this gives

$$v_0 = \sqrt{(2rL)}. \tag{2}$$

The corresponding maximum capacity is

$$c_0 = \frac{1}{p + \sqrt{(2L/r)}}. \tag{3}$$

It may be mentioned that we distinguish between absolute and relative capacity. The former concept is based on the assumption of a theoretical ***norm traffic*** moving at best speed v_0 where possible and consisting of vehicles for which the average value of L is 20 ft (vehicle length plus minimum clearance), and the maximum deceleration r on level roads is 16.6 ft/sec^2, corresponding to a useful coefficient of friction[2,3] of 0.52. *Absolute capacity* depends exclusively on the permanent physical features of the traffic channels and has become necessary and very practical as a yardstick by which to measure and compare various traffic and road conditions. The so-called ***relative capacity*** is a function of the specific conditions of traffic composition at a particular time and locality as well as of the permanent physical features of the traffic channel. Therefore it gives the actual capacity of the highway considered for the particular traffic combination it has to discharge under average weather conditions.

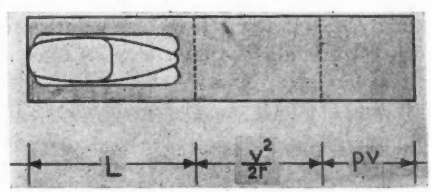

FIG. 2. Influence space, rectangular approximation.

[2] Moyer, "Skidding characteristics of tires on roadway surfaces and their relation to highway safety," Bull. 120, Iowa Engineering Experiment Station (Iowa State Coll., 1934), esp. p. 72.

[3] Hamilton, "Research on stopping distance and brake condition of automotive vehicles," U. S. Works Progress Administration for the City of New York (1935; ed. 2, 1939).

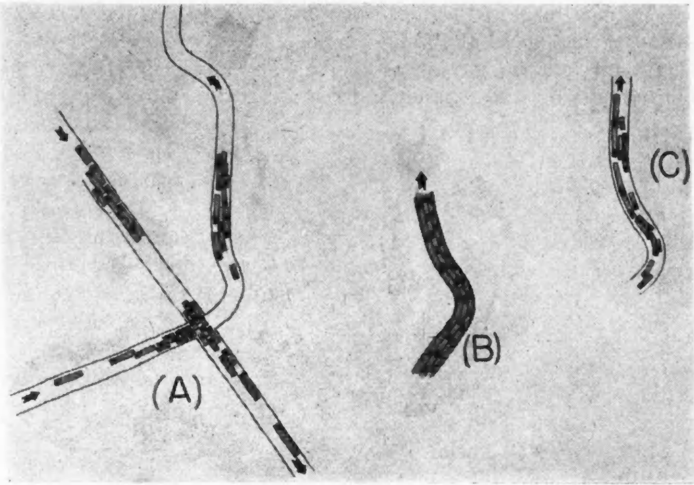

FIG. 3. Stream of influence spaces moving through road system: (*A*) clusters of vehicles near an intersection; (*B*) highway section filled to capacity; (*C*) dangerous procedure with frequently overlapping influence spaces.

If we record the progress of a moving vehicle by plotting the positions of front and rear ends of its influence space in a time-way chart, we get two time-way curves for each vehicle. The area between these two curves in the chart representing the time and space on the road required by the vehicle has a very characteristic shape. We call this area the ***influence stripe*** of the vehicle considered. Figure 4 shows influence stripes for motion at various constant speeds. Measured parallel to the time-axis the influence stripe is narrowest if the vehicle proceeds at best speed v_0 (17.6 mi/hr for norm traffic) or when capacity is a maximum.

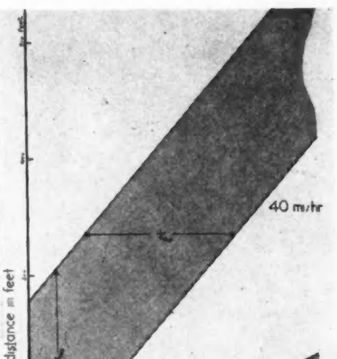

FIG. 4. Influence stripes of vehicles moving with various constant speeds.

Figures 5–7 show influence stripes during acceleration, deceleration and stopping. Here it has been taken into account that for an accelerating vehicle the influence length corresponding to the instantaneous speed v is a certain small amount Δl greater than the influence length of a car moving with constant speed v, while for a decelerating vehicle it is smaller.[4]

[4] For a car moving with constant acceleration a, the distance traveled during the perception and reaction time p is $vp+\frac{1}{2}ap^2$, where v is the speed at the moment when the obstruction appears or when the perception time begins to run. The speed after time p, or at the moment when the application of the brakes actually begins, is $v+ap$, and the braking distance required to bring the car to rest is $\frac{1}{2}(v+ap)^2/r$. Therefore, by comparison with the expression for l from Eq. (1), $\Delta l=\frac{1}{2}ap^2+apv/r+\frac{1}{2}a^2p^2/r$. The first and the third terms are very small, but the second term is not negligible for higher speeds. During the time p preceding acceleration the influence length increases steadily from l to $l+\Delta l$, and during the time p before constant speed is resumed it gradually changes back to the value l referring to motion at constant speed and given by Eq. (1). Similar considerations show that during deceleration there is a decrease in influence length which is given by the same expression Δl, if the deceleration rate (with negative sign) is substituted for a. For deceleration at maximum possible rate r, the expression for Δl reduces to $-pv$.

The broken lines in Figs. 5–7 indicate for any instantaneous speed the value of the influence length for uniform motion at the same speed. During braking with maximum deceleration the path of the front of the influence space is represented by a straight horizontal line parallel to the time-axis; this means that the influence space contracts with its front remaining stationary behind the obstruction (as required by the definition of influence space), shown in Fig. 7.

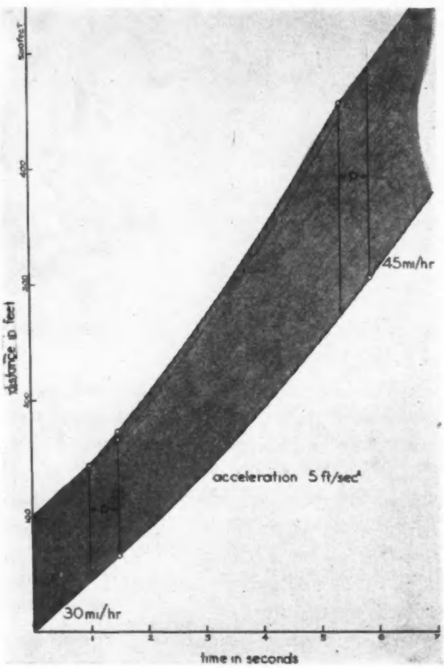

FIG. 5. Influence stripe of an accelerating vehicle.

If for a certain time interval and highway section the influence stripes of all vehicles are plotted, we get a characteristic pattern representing the performance of the traffic stream. Figures 8–10 show *elemental phases* of the stream: constant speed, acceleration and stopping in *norm procedures*. A norm procedure is obtained by fitting the influence stripes together as closely as possible without overlapping. Such procedure complies with the conditions of economy and safety. Although what actually happens on the road conforms exactly to a norm procedure only for very short periods, this is by no means merely a theoretical concept, because these periods of norm procedure continually recur in ordinary traffic. They are conspicuous on the road and particularly at intersections in those clusters of bunched-up vehicles that are so characteristic of road traffic.

The special importance of the concept of norm procedure is that such procedures serve as systems of reference for the apparently limitless variety of happenings in a field, where otherwise it would be hardly possible to create an order for a productive scientific approach. We use them as clearly defined norms for comparison with all procedures on the highway, which vary considerably through accidental circumstances, yet conform in type to the few norm procedures.

Figure 8 shows the influence stripes of a number of vehicles traveling with the same constant speed. The stripes just touch one another, which indicates that the vehicles travel in the most economical manner, utilizing road space to its capacity. If the vehicles travel at

FIG. 6. Influence stripe of a decelerating vehicle.

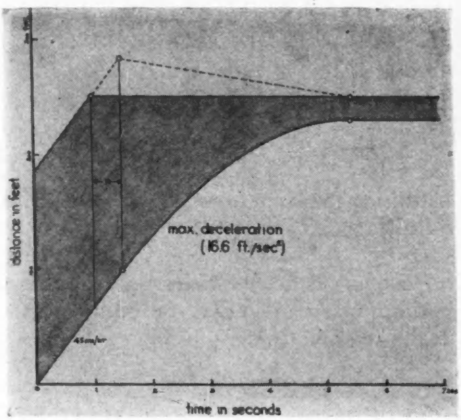

FIG. 7. Influence stripe of a vehicle stopping with maximum deceleration.

best speed $v_0[=\sqrt{(2rL)}]$, the segment cut off by the stream on any line parallel to the time-axis is a minimum, or the capacity is at its maximum, c_0. The time spacings between succeeding vehicles are a minimum, given by

$$t_0=1/c_0 \tag{4}$$

or, substituting from Eq. (3) by

$$t_0=p+\sqrt{(2L/r)}. \tag{5}$$

Figures 9 and 9(a) deal with another elemental phase in norm procedure, namely, acceleration of the stream. In Fig. 9(a) the influence stripes of the first two vehicles are shown in detail for acceleration from rest to some speed v. At first both influence stripes are parallel to the time-axis, their width being equal to L (vehicle length plus minimum distance between stationary vehicles). The first vehicle then starts to accelerate at the rate a_1, with a corresponding increase of its influence length until it has attained the speed v. Then it proceeds with constant speed v, its influence stripe being now straight with the slope v and the width parallel to the distance-axis equal to the influence length l corresponding to constant speed v. The succeeding vehicle starts to accelerate a little later than the first, let us say, t_2 sec later. Since its driver of course cannot foresee at what speed v the preceding vehicle will cease to accelerate, he himself can at best accelerate at such a rate a_2 that, at the moment when his car has attained speed v, its distance from the vehicle ahead will be exactly equal to l, so that it can then trail it as closely as possible at constant speed v. Otherwise he might find himself forced at that—unpredictable—moment to brake suddenly at maximum rate so as to prevent a collision.

If the foregoing procedure is possible, then t_2 and a_2 must have such values as to make $IH=l$. But $IH=IG-HG$, and $IG=v\cdot KG$. Therefore, the condition for t_2 and a_2 is

$$v\cdot KG-HG=l.$$

Substituting

$$KG=AB+BD-AC=t_2+v/a_2-v/a_1, \tag{6}$$

$$HG=HD-GE-ED=\tfrac{1}{2}v^2/a_2-\tfrac{1}{2}v^2/a_1-L, \tag{7}$$

and l from Eq. (1), and rearranging terms, we get

$$v^2\left(\frac{1}{2r}-\frac{1}{2a_2}+\frac{1}{2a_1}\right)+v(p-t_2)=0.$$

Since the value of v is not known when the acceleration of the second vehicle begins, this equation should hold for any value of v. Therefore, both expressions in parentheses must be zero, or

$$\frac{1}{a_2}=\frac{1}{a_1}+\frac{1}{r},\quad t_2=p. \tag{8}$$

In the same manner we find for the time interval t_n between the starting of the $(n-1)$th and nth vehicles,

$$\frac{1}{a_n}=\frac{1}{a_{n-1}}+\frac{1}{r},$$

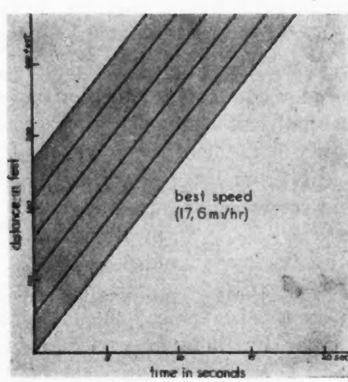

FIG. 8. Stream movement, elemental phase of constant speed in norm procedure.

or, after substituting successively

$$\frac{1}{a_{n-1}} = \frac{1}{a_{n-2}} + \frac{1}{r},$$

and so forth,

$$\frac{1}{a_n} = \frac{1}{a_{n-1}} + \frac{n-1}{r}. \tag{9}$$

These equations characterize the elemental phase of acceleration from rest in norm procedure. The traffic stream is represented by a family of parabolas. For successive vehicles their half-parameters increase by equal increments $1/r$, and their vertices are shifted by equal amounts, p (sec) to the right and L(ft) downward. Thus they are a projection of the intersections of a cone with a family of equidistant planes parallel to the generating line. Such a family of time-way curves shows some interesting properties.

Consider, for instance, the points K and H corresponding to the same speed v on the first and second curve; their distance apart KH is given by

$$KH = \sqrt{(HG^2 + KG^2)},$$

and the slope of KH is given by

$$\tan\theta = HG/KG. \tag{10}$$

But from Eqs. (6)—(8),

$$HG = \tfrac{1}{2}v^2/r - L, \tag{11}$$

$$KG = v/r + p. \tag{12}$$

Since both HG and KG and therefore also KH and $\tan\theta$ depend only on the value of v and the general constants r, L, p, they must have the same value for any two successive vehicles; or, for all vehicles, the points corresponding to the same speed are equidistant points on one straight line. We shall call such a line an *isospeed line*. Substituting in the last two equations for v the best speed v_0 from Eq. (2), we find for the isospeed line corresponding to best speed, or the *best speed line*, $H_0G_0 = 0$, meaning that the best speed line is horizontal, and $K_0G_0 = \sqrt{(2L/r)} + p$, or from Eq. (5), $K_0G_0 = t_0$, meaning that the time spacings on the best speed line are minimum time spacings. Furthermore—referring as origin to the point O, when and where the first vehicle starts (rear end)—the coordinates τ, σ of the point of intersection P of the best speed line with an isospeed line corresponding to some speed v are $\tau = -PL$ and $\sigma = OL$, where $OL = \tfrac{1}{2}v_0^2/a_1$, or, from Eq. (2),

$$OL = rL/a_1$$

and

$$PL = PM - LM = KM/\tan\theta - LM.$$

Substituting for $\tan\theta$ from Eqs. (10)—(12), and substituting $KM = \tfrac{1}{2}(v^2 - v_0^2)/a_1$ and $LM = v/a_1$, we get $PL = rp/a_1$. Therefore, since neither OL nor PL depends on v, *all* isospeed lines intersect the best speed line at the same point P with the coordinates

$$\sigma = rL/a_1, \tag{13}$$

$$\tau = -rp/a_1. \tag{14}$$

We can now easily determine the norm procedure of the traffic stream for acceleration from any initial speed v_i to any final speed v_f. It is given

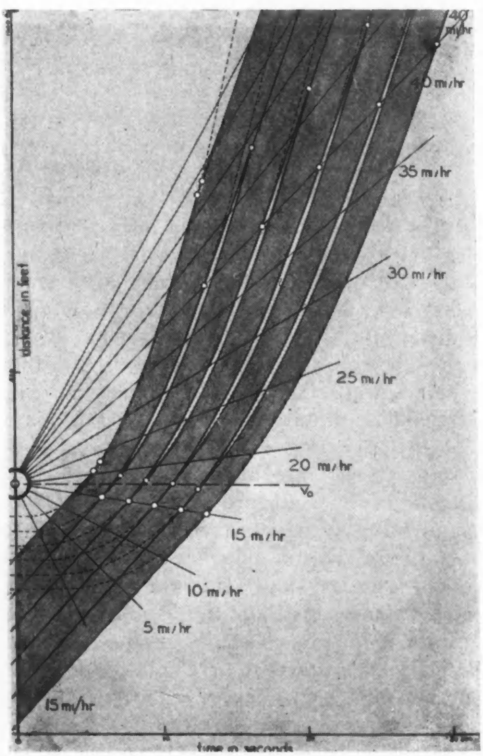

FIG. 9. Stream movement, elemental phase of acceleration in norm procedure.

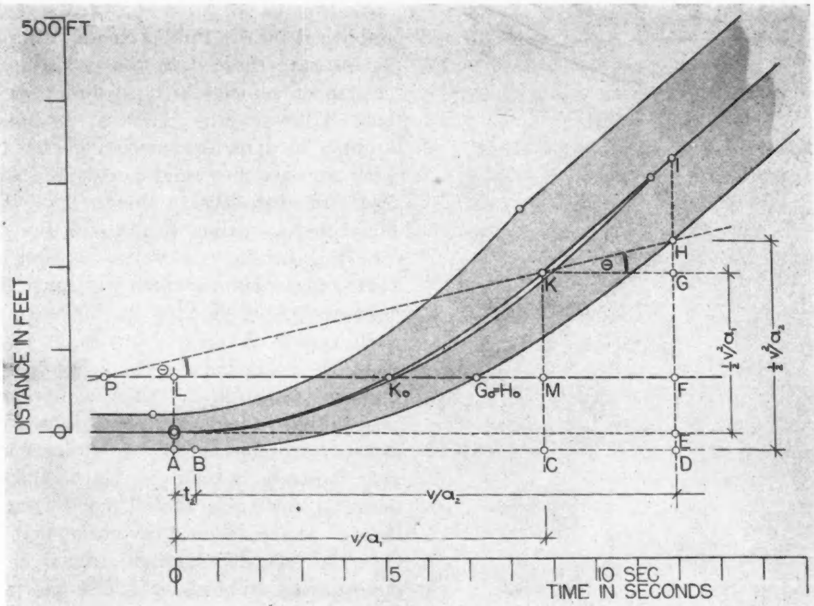

FIG. 9(a). Detail of influence stripes of the first two of a group of vehicles accelerating from rest to some speed v in norm procedure.

by the segments that the isospeed lines corresponding to v_i and v_f cut out of the same family of parabolas. If the stream before and after acceleration moves with constant speed, it is then represented by the parallel straight lines tangent to the parabolas at the points where they intersect these two isospeed lines. This is shown in Fig. 9 for change in speed from 15 mi/hr to 40 mi/hr.

It will be noted on Figs. 9 and 9(a) that during the norm procedure of acceleration the influence stripes do not touch but are separated by narrow gaps. These gaps appear p sec before acceleration starts and disappear p sec before constant speed is resumed.

Figure 10 shows the norm procedure for the stopping performance of the stream. The first vehicle decelerates at a rate r_1, which in the case shown is taken as the maximum possible braking rate r. The second vehicle begins to decelerate t_2 sec later and at the rate r_2, where t_2 and r_2 must have such values that the vehicle comes to rest exactly L ft behind the first vehicle. Applying the same method as before, we find expressions for t_2 and r_2. Then we obtain quite generally for t_n and r_n:

$$t_n = p, \quad \frac{1}{r_n} = \frac{1}{r_{n-1}} + \frac{1}{r},$$

or, by successive substitution,

$$\frac{1}{r_n} = \frac{1}{r_1} + \frac{n-1}{r}.$$

If, as in the case shown in Fig. 10, the first vehicle stops at maximum rate ($r_1 = r$), then

$$r_n = r/n. \tag{15}$$

Again there are isospeed lines intersecting at one point, and if, especially, $r_1 = r$, this is just the point where the line representing the path of the front of the influence space of the first vehicle becomes horizontal. During deceleration, too, gaps appear between the influence stripes of the successive vehicles.

Figure 11 shows the norm procedure for a composite motion of the traffic stream consisting of six phases: the first vehicle decelerates at maximum rate from an initial speed v_i (here 30 mi/hr)

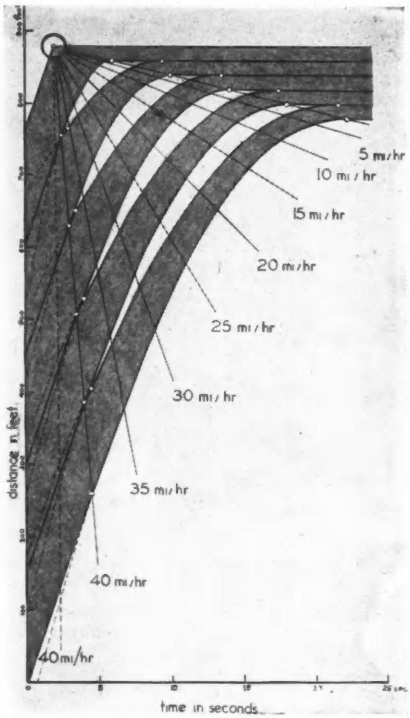

FIG. 10. Stream movement, elemental phase of maximum deceleration in norm procedure (stopping performance).

to v_f (here 15 mi/hr), proceeds at constant speed v_f and then accelerates until it has again attained the speed v_i. The succeeding vehicle starts to decelerate p sec later at the rate $r_2=\frac{1}{2}r$, according to Eq. (15), but p sec after the first vehicle has ceased to decelerate it reduces its deceleration so as to attain speed v_f at the distance of one influence length from the first car. Then it proceeds at constant speed v_f until, $p+v_f/r$ sec later than the preceding car, it starts to accelerate again to speed v_i. The same consideration applies to all succeeding cars so that we can distinguish six phases in the motion of the stream: **(A)** constant speed v_i; **(B)** deceleration at rate $r_n=r/n$; **(C)** deceleration at reduced rate

$$r_n'=\frac{r_n(n-1)}{n(v_i+v_f)/(v_i-v_f)-1};$$

(D) constant speed v_f; **(E)** acceleration at rate a_n given by Eq. (9); **(F)** constant speed v_i.

In Fig. 12 we have analyzed some field data published by the Public Roads Administration.[5] To procure these data the procedure of a succession of vehicles in both directions of a two-lane highway was recorded by means of a number of detectors consisting of rubber tubes with air switches, laid across the highway at 50-ft intervals over a distance of 0.5 mi and connected by cables to graphic time recorders.[6] The diagram shows a typical example of an overtaking performance which we transcribed into a time-way chart showing the influence stripes for each vehicle. Vehicle *3* overtakes vehicle *2* by using the space between vehicles *4* and *5* on the opposing lane. We see that vehicles *1*, *2* and *3* proceed with their influence stripes touching but not overlapping before the process starts and after it has been achieved. Only vehicle *3*, in its desire to overtake, though conditions for overtaking are actually not favorable, first comes too close to vehicle *2*, then encroaches on the opposing lane before vehicle *4* has passed sufficiently. Then it cuts in too close in front of *2* and too close behind vehicle *1*.

Figure 13 is based on the actual record of a section of the traffic stream consisting of a typical combination of passenger cars, trucks and truck semi-trailers moving on a 6-percent gradient of a two-lane highway.[7] Since the hill-climbing ability of heavy trucks is very limited, they force all succeeding vehicles to slow down. The passenger cars try to overtake the trucks whenever the opposing traffic lane is free. It is interesting to note that practically all vehicles move according to the safe and economic norm procedure. There are some careless drivers—No. 7 and 16—who repeatedly deviate from this procedure, but they correct these deviations immediately. Others, more experienced, are able to keep to it instinctively even under difficult circumstances. Vehicle *20* on the opposing lane provides an example for the assertion that at very high speeds the necessary influence space is often underestimated, an idea also corroborated by

[5] Norman, Public Roads **20**, 221 (1940).
[6] Holmes, Public Roads **19**, No. 11 (1939); Thompson, "Studies of driver behavior and vehicle performance by U. S. Public Roads Administration," paper presented at Johns Hopkins University (Oct. 27, 1939).
[7] U. S. Route 66 near Edwardsville, Ill. on Oct. 4, 1939; see F. N. Barker, Public Roads **20**, 228 (1940).

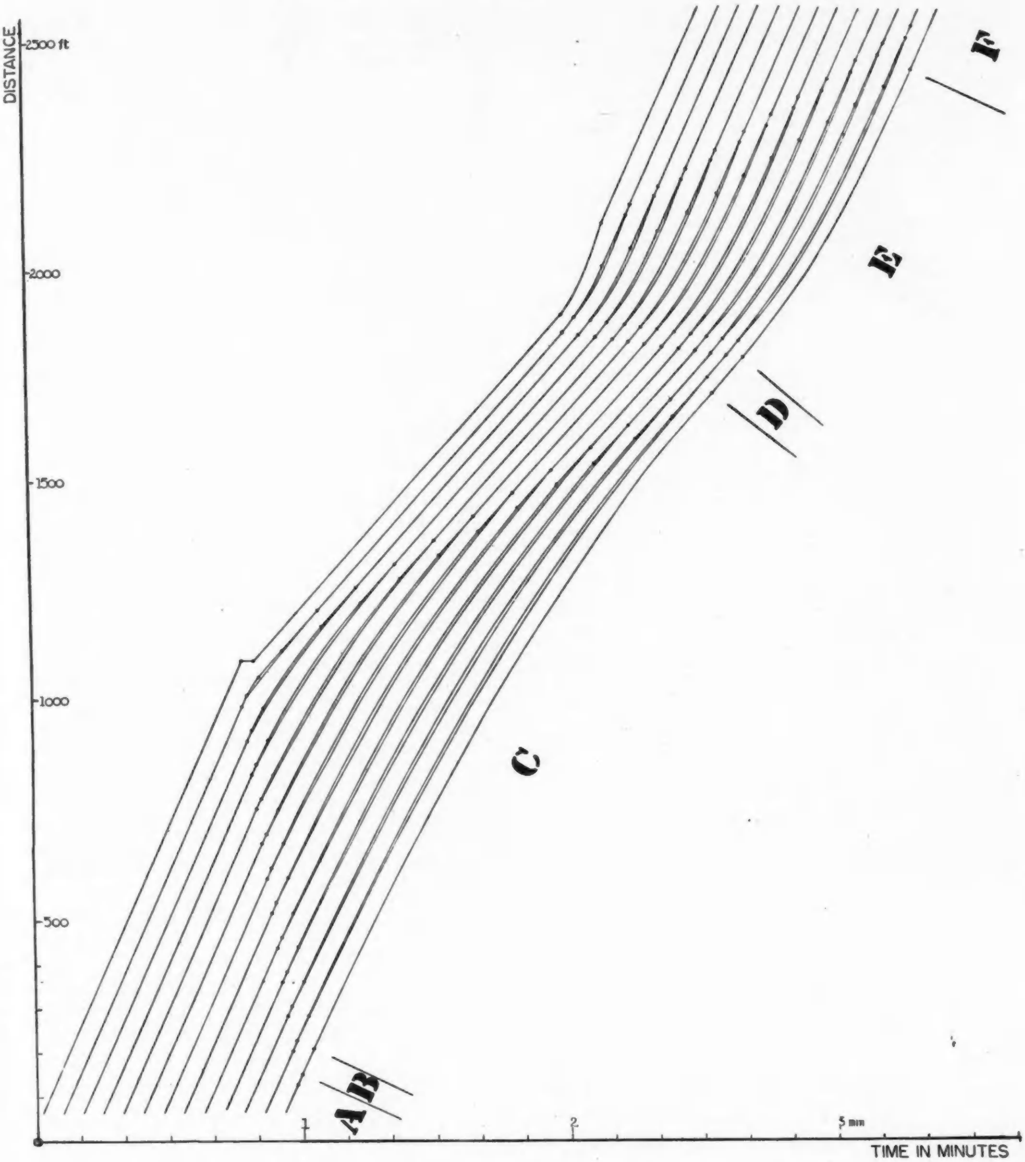

FIG. 11. Example of a composite motion of the stream in norm procedures. Phases: **A**, constant speed (30 mi/hr); **B**, deceleration at rate r_n; **C**, deceleration at reduced rate r_n'; **D**, constant speed (15 mi/hr); **E**, acceleration at rate a_n; **F**, constant speed (30 mi/hr).

other data compiled by the Public Roads Administration. The usual argument in defense of such driving is that at high speeds and consequently greater distances between vehicles there will always be space enough to avoid collision by steering out of the difficulty. But Chapman[1] has shown that turning in most cases is less efficient than the straight stop to avoid collisions.

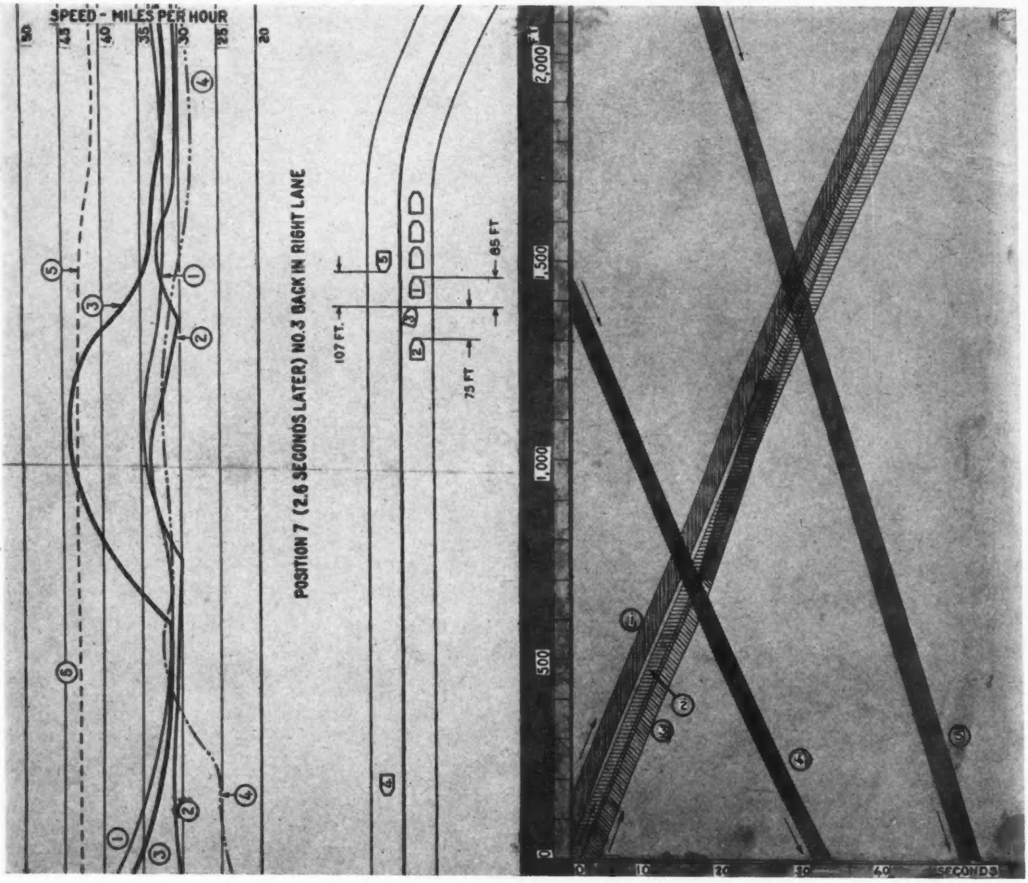

FIG. 12. Overtaking procedure on a two-lane highway: *left*, recordings of data taken by Public Roads Administration; *right*, analysis in time-way chart of influence stripes.

The paramount fact apparent from the examination of such charts is this: *Even where vehicles proceed with overlapping influence spaces and however dangerous this may become in a specific instance, the possible deviation from the norm procedure is so negligible that it does not change the characteristic pattern of each phase of the stream.* This is conspicuous when we compare Fig. 13 with Fig. 11 and also with Fig. 14. The latter shows another time-way chart of a norm procedure, this time for the most significant case, namely, at a typical intersection. Let us assume that the vehicles of the stream are lined up behind the red stop light and that the light changes to green. After the time p the first car accelerates at maximum rate (the value used here is 5 ft/sec^2, representing about the maximum possible acceleration for average passenger vehicles[8] to the desired speed, here 30 mi/hr. The succeeding cars move according to the curves derived before for the elemental phase of acceleration until they also have the speed 30 mi/hr. Now the significance of the best-speed line becomes apparent: we see that there is on the road a line beyond the stop line which all vehicles pass at the same speed and with equal

[8] Compare Loutzenheiser, Proc. Highway Research Board **18**, 90 (1938).

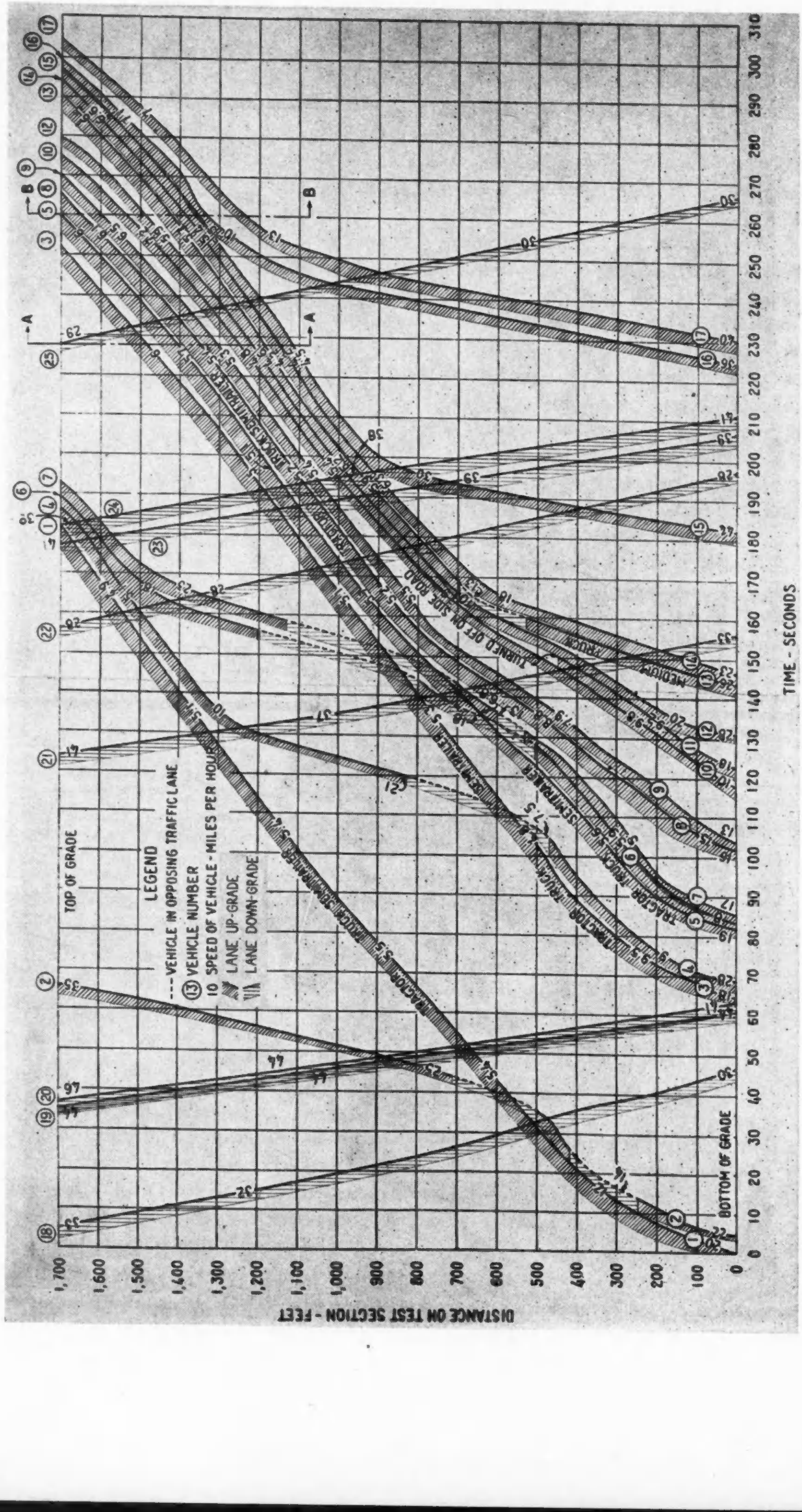

FIG. 13. Analysis of data on motion of traffic stream on a two-lane highway with 6-percent gradient in time-way chart of influence stripes. (Data recorded by Illinois Division of Highways.)

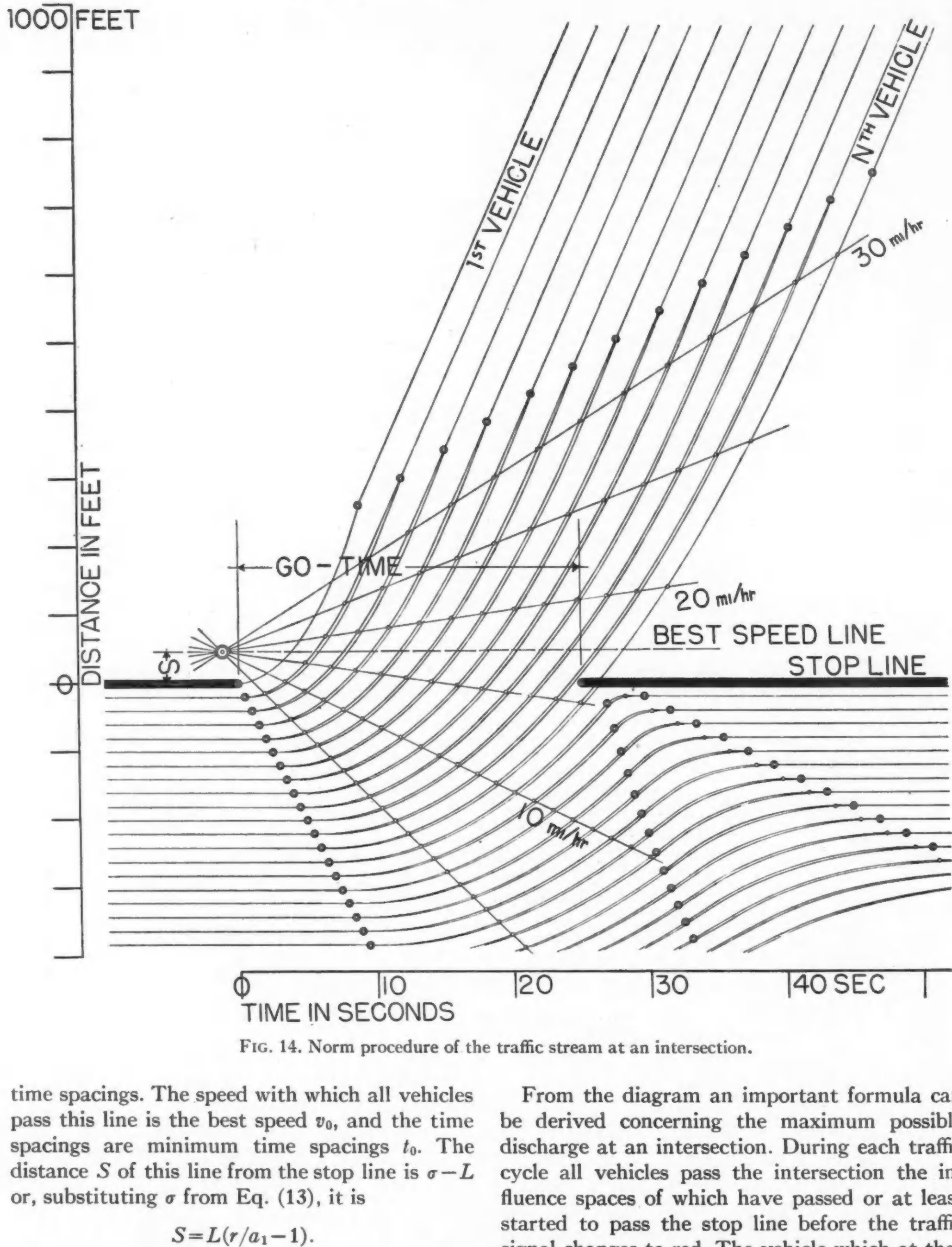

FIG. 14. Norm procedure of the traffic stream at an intersection.

time spacings. The speed with which all vehicles pass this line is the best speed v_0, and the time spacings are minimum time spacings t_0. The distance S of this line from the stop line is $\sigma - L$ or, substituting σ from Eq. (13), it is

$$S = L(r/a_1 - 1).$$

For norm traffic S is 46 ft.

From the diagram an important formula can be derived concerning the maximum possible discharge at an intersection. During each traffic cycle all vehicles pass the intersection the influence spaces of which have passed or at least started to pass the stop line before the traffic signal changes to red. The vehicle which at that moment is still in front of the stop line will yet

pass if its influence space overlaps the stop line, because—according to the definition of influence length—it cannot stop clear of the stop line. This, of course, is limited to a single vehicle in each lane. Hence, the maximum number n of vehicles that can be discharged per lane is determined by equating the go-time t_g to the time when the rear of the $(n-1)$th vehicle passes the stop line. Since the $(n-1)$th vehicle starts at time $(n-1)p$ after t_g begins to run and from a distance $(n-1)L$ behind the stop line,

$$t_g=(n-1)p+\left[\frac{2(n-1)L}{a_{n-1}}\right]^{1/2},$$

or, in view of Eq. (9),

$$t_g=(n-1)p+(n-1)\sqrt{(2L/r)}\left[1+\frac{r-a_1}{(n-1)a_1}\right]^{1/2}.$$

Upon expansion of the last square root this becomes

$$t_g=(n-1)p+(n-1)\sqrt{(2L/r)}+\sqrt{(2L/r)}\frac{r-a_1}{2a_1},$$

or, from Eq. (5),

$$t_g=(n-1)t_0+\sqrt{(2L/r)}\frac{r-a_1}{2a_1},\ \text{approximately.}$$

Now for all practical values of L, r, a_1 and t_0, the second term happens to be very nearly equal to t_0 so that, as a very close approximation,

$$t_g=nt_0. \tag{16}$$

If we remember that the time spacings on the best-speed line are just equal to t_0, we see from this equation that the segment cut off by the stream on the best-speed line should be about equal to the go-time. This can easily be verified directly from the diagram.

Since n is the maximum number of vehicles that can pass during the whole cycle-time t_c, we find that the capacity c at an intersection is n/t_c vehicles per second, per lane; or, from Eqs. (16) and (4),

$$c=c_0t_g/t_c,\ \text{approximately.} \tag{17}$$

It can readily be shown that for norm traffic and for go-times exceeding 30 sec, Eqs. (16) and (17) are correct to less than 1 percent. For shorter go-times the error is slightly greater, but even if the go-time were as short as 15 sec it would still be less than 3 percent.

Formula (17) for the capacity at an intersection was derived on the assumption that all vehicles arrive at the intersection when the signal is red and therefore have to stop and line up behind the stop line. We see, however, that the resulting capacity represents an optimum value since we would get the same value if all vehicles moved with best speed v_0 and arrived at the intersection when the signal is green so that they could proceed unhindered. This proves that the capacity loss at an intersection is owing solely to the temporary closing of the road and that there is no "acceleration loss," as was assumed in another investigation.[9] On the other hand, for the same reason, the capacity of a highway with many intersections cannot be increased by introducing the so-called progressive traffic-signal system, where signals are timed with respect to each other so as to allow undelayed motion at a planned speed, beneficial as this system of course is to the individual driver.

The total capacity C of two intersecting lanes is, from Eq. (17),

$$C=c_0(t_c-t_w)/t_c,$$

where t_w represents the sum of the two warning times per cycle when both roads must be closed simultaneously at the change from red to green. Hence the capacity of an intersection increases with increasing cycle time. Figure 15 shows the combined capacity of two intersecting highways in percentage of the capacity which they would have if one were overpassing the other, and the corresponding average delay as functions of the cycle time (we assumed $t_w=10$ sec). It should be noted that—while the cycle time cannot well be under 30 sec—under normal conditions, increase of cycle time to more than about 100 sec does not produce any appreciable gain in capacity. There are also very definite limits set to the advantage of increasing the cycle time, because the average delay increases in direct proportion to the cycle time and therefore at a much higher rate than the capacity of the intersection.

[9] Compare *Proceedings, Institute of Traffic Engineers* (1934), pp. 19–33.

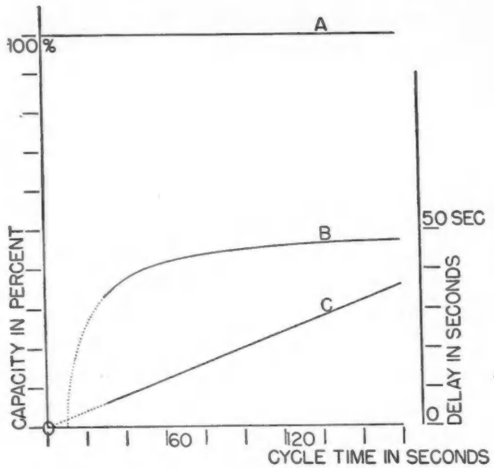

FIG. 15. Capacity (curve B) of two intersecting highways in percentage of their combined capacity if one overpasses the other (curve A), and average delay (curve C) at the intersection, as functions of the cycle time. Scale to the left refers to curves A and B; scale to the right, to curve C.

Other practical applications of this study concern, first of all, general methods of measuring and comparing the efficiency of any highways in terms of capacity. The influence of single physical features of the traffic channel, such as curves, gradients, clear-sight distances and surface conditions, and also features of the traffic itself, such as its average speed, vehicle types and performance, and braking power under various weather conditions, can be determined. This makes it possible to establish in terms of absolute and relative capacity the efficiency of highways as regards traffic volumes, possible or practical speeds, economy and safety of discharge on the basis merely of specifications or of large-scale maps. Up to now this has been done by a rule-of-thumb method through comparison with sample highways on which the traffic has been counted. It is obvious that objective criterions for efficiency and performance of highways and traffic are necessary not only for such special purposes but also generally for the more important investigations of the means of planning, design and traffic control to be employed for individual highways and whole road systems.

These applications, however, are only side lines of this study. It was soon realized that the consideration of single factors and features had to be integrated into studies of the stream as a whole and of the continuous system of channels through which it flows like a liquid. We encounter more intricate conditions as soon as we take into account the variations in the width of the influence space and correct the rectangular approximation in the light of more detailed knowledge of the conditions which determine its dimensions. A good deal more experimental research will be necessary on maximum and average acceleration and deceleration in various circumstances, notably of speeds and gears, and on the influence of speed and gear on turning times and angles, friction factors and steadiness of course, before the more complex problems of the physics of highway traffic can be solved.

A trained, and mind you I say trained, scientific researcher thinks only of the object he has before him, not of any ideology, not of himself, not of his publicity, not of what anybody thinks of him or his associates, not of another job—but only of one thing—what do the facts justify? How helpful it would be if we could have more trained minds to see errors, to pass judgment and guide action before it is too late. —BERNARD M. BARUCH.

Pictures and Images

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NOT many people can state correctly, without using the words "right" and "left," what is the difference between their own two hands. That difference is fundamental in geometrical optics, and in other parts of physics also, and it is not clearly or adequately treated in any textbook that I have seen. The right hand differs from the left (assuming no accidental differences) because the order or sequence of corresponding particles in one is reversed in one dimension with respect to the corresponding sequence in the other. Let the reader hold his two hands before him, fingers pointing forward, thumbs up, palms together. There is a one-to-one correspondence of particles, but those of the left hand are in order opposite to those of the right, along parallel horizontal lines perpendicular to the palms, or perpendicular to the plane of symmetry between the two hands. A little thought will show that, with both hands spread or with both fists clenched, the parallel lines along which reversal appears have no preferred direction with respect to the fingers, or with respect to the palms. In fact, if any set of parallel lines is drawn through the right hand (for example) and an image is constructed by arranging particles along those lines to correspond with the particles of that hand, but in reverse order along each line, the image is a left hand. However, it is possible, and easy, to turn one hand or the other around until the two are opposite or reversed in three dimensions. We may hold the right hand with fingers pointing south, thumb up and palm to the east; then the left hand may be turned so that fingers are toward the north, thumb down, and palm to the west, being then opposite to the other in three (mutually perpendicular) directions. But the two hands cannot be so placed that they are opposite in two dimensions only. If two hands are opposite in two dimensions they are both right (or both left) hands.

Now a plane mirror produces an image reversed with respect to the object in one dimension—namely, the dimension perpendicular to the mirror. One may hold his right hand before a vertical mirror with fingers pointing toward the mirror. In the mirror the direction of pointing is reversed, but if the actual thumb points upward, so does the image thumb point, and if the back of the hand is toward the west (observer facing south) the back of the image hand is toward the west also. If one stands upon or beneath a horizontal mirror, his image is inverted, but not reversed horizontally. Whichever dimension of the object is perpendicular to the mirror is subject to the reversal, and that dimension only. This follows obviously, and inescapably, from the oft-repeated proposition that the image of a point is as far behind the mirror as the point is in front, and on the same normal to the mirror. But necessary and important consequences of that proposition are often neglected and ignored.

It is worthy of remark that two screws, or two helixes, may differ as the two hands differ, so that one is the mirror image of the other. It is conventional to call one of them a right-hand screw, but it is clear that there is no reason except convention for calling that one right-hand rather than the other. There is no rational necessity for the conventional association. A rectangular coordinate system, also, having the positive x-, y- and z-axes parallel to the thumb, index finger and middle finger, respectively, of the right hand (the middle finger being perpendicular to the palm), is called a right-hand set. If one axis is reversed the coordinate system becomes a left-hand set. The difference between them is indeed the difference between the two hands, but the right-hand set is not intrinsically right. It is right by convention only.

The reversing effect of reflection is not confined to reflection by a plane mirror. The image of a right hand (forming either a real or a virtual object) produced by a spherical mirror, whether concave or convex, is a left hand. This may be checked by trial, and it may be proved algebraically. To this proof we proceed, but it is necessary first to set up the sign conventions that will be followed in the proof.

The light in all situations is to come from the left and go toward the right to the mirror or lens. With mirrors the object distance p and the image distance q have the same sign when object and image are on the same side, both positive when measured to the left. With lenses p is positive when the object, which is real, is on the left, q is positive when the image is on the right, being then a real image. If the object is on the right it is virtual, and p is then negative. Other coordinates, x and y, mean object distance and image distance also, but with a mirror x and y are measured from the principal focus F, both positive if to the left from F, whether the mirror is concave or convex. With a lens there are two foci, F and F': F is to the right of a converging lens and F' to the left; F is to the left of a diverging lens and F' to the right; x is measured from F', positive if to the left, y is measured from F, positive if to the right. All these conventions are represented in Fig. 1, which gives also the classification of objects as real or virtual. But it is not to be inferred from this figure that q is negative if p is negative, nor that q is positive if p is positive. Virtual objects are commonly ignored in elementary accounts, but they may be involved whenever two lenses, two mirrors or a lens and a mirror are in use.[1] A virtual object may be realized if light converges toward a point beyond a lens or mirror. The point to which it would converge, if the lens or mirror were not there, is the virtual object.

The object distance p and image distance q are connected by the well-known formula $(1/p)+(1/q)=1/f$, and x and y by the still simpler equation $xy=f^2$. Since the latter equation is the easier to use, and since the sign of y is always the same as that of x, it is remarkable that this equation is not more commonly taught. The transformation from one of these equations to the other is a perfect example of a change of variable, or of coordinates, and of the remarkable simplification that may result from such a change, and it is interesting on its own account. One frequently encounters the statement that a diverging lens (or a plane or convex mirror) will not produce a real image. Yet a diverging lens, for example, does form a real image if the object is virtual and at a distance from the lens numerically less than the focal length f. It is certainly more economical and safer to use the equations with signs positive as given for all cases, and to substitute for the letters the given numbers with their proper signs, thus automatically obtaining the proper sign with the numerical answer, than to change the signs in the general equation to suit the particular situation in hand.

To prove now that the image of a right hand formed by a mirror is in all cases a left hand requires only that one differentiate the terms in

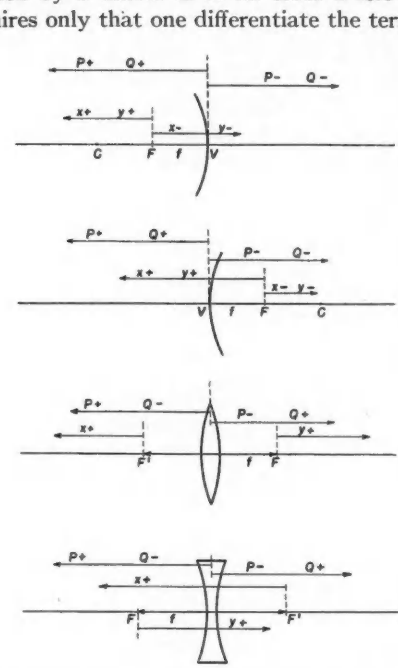

FIG. 1. Sign conventions for mirrors and lenses.

the general equation, for example, with respect to p. This gives directly

$$-1/p^2-(1/q^2)\mathrm{d}q/\mathrm{d}p=0,$$

or

$$\mathrm{d}q/\mathrm{d}p=-q^2/p^2=-(q/p)^2.$$

Evidently $\mathrm{d}q/\mathrm{d}p$ is negative, no matter what signs q and p may have. Since for small displacements of object and image, $\Delta q=(\mathrm{d}q/\mathrm{d}p)\Delta p$, approximately, Δq represents a decrease in q if Δp is an increase in p. A positive Δp, or an increase in p, means that the object moves to the left, and

[1] For example, see L. M. Alexander, Am. J. Phys. **10**, 110 (1942).

Δq, or the resulting change in q, is then negative, meaning that the image moves to the right. Hence, whenever the object moves, the image moves the other way. This is a general rule for plane and spherical mirrors and true throughout the whole range of values.

Now if a right hand is held before the mirror, and one follows the image point as an object point moves through the hand along the axis, the image point moves oppositely, and the image is reversed with respect to the object in one dimension, namely, that parallel to the axis. The image is either reversed in two dimensions perpendicular to the axis, or it is erect and not transversely reversed at all. Hence either it is reversed in one dimension or it is reversed in three dimensions. In either case the image of a right hand is a left hand. We may draw the same conclusions from the equation in x and y, where the differentiation and interpretation make an excellent exercise.

In elementary accounts too much emphasis is placed on the inversion of the image. The conventional figures showing the construction of images represent any plane through the axis, and if the image is inverted in one such plane, so it is in any other also. It is reversed horizontally if it is reversed vertically. More properly, if the image is inverted this is because it is rotated on the axis through 180° with respect to the object. If the rays used in construction cross the axis between object and image, the image is rotated 180°. If these rays do not cross the axis between object and image there is no rotation. Therefore, there is reversal in two transverse dimensions or in none. Accordingly, consideration of the third dimension (parallel to the axis) is necessary in order to determine whether the image of a right hand is a right hand or a left, or, properly speaking, whether or not the image is perverted. To transform the right side of the object into the left side of the image may produce a perverted image, and may not, as we shall presently see in the case of lenses.

The foregoing equations in p and q, and in x and y, as well as those obtained by differentiation, hold as written for lenses as well as for mirrors. However, for lenses the sign convention is different, as shown in Fig. 1. Accordingly, Δp and Δq are necessarily of opposite sign. A positive Δp means that an object (real or virtual) moves to the left, and the image, since Δq is negative, moves to the left also. Object and image always move along the axis in the same direction. Hence, if the object point moves through the right hand toward the left, for example, the image point moves to the left also, and object and image are not reversed along the axis. Since, as with mirrors, there will be reversal in two transverse dimensions or in none, there are in all either no reversals or two. The image of a right hand formed by a lens is a right hand in all cases.

Here we may insert the remark that if a man had a third member that could properly be called a hand, it would necessarily be a right hand or a left hand. There is no other possibility; there can be only two kinds of hands.[2] Accordingly, the perverted image of a perverted image is natural with respect to the original object. In this respect to pervert corresponds to taking the negative reciprocal of a number. When that is done a second time one recovers the original number. We shall refer to an image that is not perverted as "natural."

Besides discriminating for us between perverted and natural images, our differentiation gives us also the longitudinal magnification of the image. Let $o_\perp$ and $i_\perp$ represent corresponding linear dimensions of object and image measured perpendicular to the axis. Then we have the familiar relation $i_\perp/o_\perp = -q/p$, giving the ratio of sizes, or lateral magnification, and showing that real images of real objects, for example, are inverted. If $o_\parallel$ and $i_\parallel$ are measured parallel to the axis, $i_\parallel/o_\parallel = -(q/p)^2$. This means that if the image is 10 times as large as the object, when measured across the axis, it will be 100 times as large if measured along the axis—supposing that the object is actually not large compared to the focal length of the lens or the mirror. If a small cube is placed on the axis with two faces perpendicular to the axis, the image will be a cube only if $q=p$. If $p \neq q$, the image will be approximately rectangular if the object is very small, but it will not be even approximately a cube. If $q>p$ the

[2] There are, apparently, also only two sexes. The worker bees, which ordinarily exercise no sexual functions, are undeveloped females.

image is thick, measured along the axis; if $p>q$, the image will be thin.

This longitudinal magnification, so different in general from the transverse, is not considered important, perhaps because it does not determine the size of a picture thrown upon a screen, or upon the film of a camera. But the difference between the two magnifications is easily observed. Use an 8-power binocular, for example, to look at a band 100 yd away or more, or at an orchestra from the back of a large auditorium. One will notice directly that the enlargement from side to side is much greater than the magnification in distance or in depth. The observer will expect the first or nearest violinists to elbow their neighbors next behind every time they draw back their bows. It is clear that the objective of the binocular gives a transverse reduction of the real image by the factor 200, perhaps, and hence a longitudinal reduction by the factor 40,000. The eyepiece gives a virtual image, which may be enlarged any amount; but if the binocular is focused for closest discrimination in vision, the transverse enlargement by the eyepiece is ordinarily not more than 10, and not nearly enough, even when squared, to counteract the great reduction in depth given by the objective.

This squared ratio works greatly to the advantage of the camera, or of the human eye. If one looks at a building at some moderate distance, for example, the retinal figure may be reduced transversely in the ratio 1000. In such case the image in space will be reduced longitudinally by the factor 1,000,000; hence no part of the three-dimensional image will be far from the receiving surface. The whole building, near parts and far parts, will be almost in focus at any one setting of the adjustable lens.

It is time now for the reader to perform a simple experiment. Let him hold his open left hand, palm upward, beneath his shaded lamp, and, with a common reading glass, focus the image of the illuminated left hand on the unlighted ceiling. He may then prepare to enter a general demurrer, for, observing the projected picture, he will conclude that what he has been reading is entirely wrong. The figure on the ceiling is reversed horizontally in two dimensions, or rotated 180° on a vertical axis, and the palm is down. Accordingly it is reversed in three dimensions with respect to the object, and it seems to be, and must be, a right hand. Yet we have just claimed to prove that the image of a left hand formed by a lens is a left hand. In order to explain this absolute contradiction (as it now must seem) we must make an important distinction which is inadequately treated or entirely ignored in every textbook with which I am familiar.

In considering longitudinal magnification and reversal along the axis of a mirror or lens we have been thinking of an actual object with three dimensions, and of a three-dimensional image. With mirrors and lenses (actually, of course, with spherical surfaces only if the aperture is small) the points in image space correspond, one for one, with the points in object space. The three dimensions of an image must all be taken into account in order that one may say whether the image is perverted or natural. In this sense of the word a small opening, as used in a pinhole camera, does not produce an image. There is no point in space corresponding uniquely to a particular point of the object. One cannot observe this "image" in space, without a screen, by looking directly at it or by focusing upon it with an eyepiece. There being no image, it is idle to declare that the image is, or is not, perverted. However, many books begin the discussion of mirrors and lenses with a section on "Images produced by small apertures." After such a start, understanding is impossible and the student's case is hopeless.

The pictures of brightly illuminated objects formed on a screen by light passing through a small opening are familiar to everyone. Let the screen be translucent, like ordinary tracing cloth, so that the picture may be seen from either side. Let the illuminated object be a right hand, fingers up, thumb to the west and palm to the north and toward the opening. If the picture is viewed from the south side, as one looks away from the hole, it seems to be a left hand. If viewed from the north (from the back of the screen), so that the beholder looks toward the opening, the picture represents a right hand. With respect to the object it is reversed in two dimensions only, since the palm is toward the north. An actual right hand, however, is a right hand, and seems to be a right hand, no matter how one may behold it. A two-dimensional

picture of this kind I prefer not to call an image. The term *image* may better be reserved for a three-dimensional manifold, the points of which show a one-to-one correspondence with those of an object. At any rate, the difference between an image in space and a two-dimensional pattern on an opaque surface is an important difference, and it is worthy of emphasis by a verbal distinction. Moreover, this distinction is maintained in common language, where an image is a three-dimensional object, and a representation in two dimensions is a picture. Only in physics, where the difference is vital, is it obscured by negligence and ambiguity.

If the reader will now prop up a firm right-hand glove beneath his shaded lamp, and with a reading glass form the picture on a sheet of thin paper, he can check these assertions. The paper will show a right-hand glove if viewed from the back. Also if no paper is used, but the beholder looks directly at the real image, his eyes being necessarily within the cones of light that diverge from its several points, he will see a right-hand glove. Even as formed by a lens the flat picture is not an image, there being no unique correspondence of points between it and the object.

If it were possible for one (with proper equipment) to look into the eye of another person, and, when the person being examined is looking at an illuminated right hand, for the examiner to see objectively the image on the retina, it would seem to the examiner to be a left hand. Yet the subject—the person being examined—recognizes the hand without question as right. It seems that there is something harder than inversion which the mind must correct or interpret when it acquires verifiable knowledge of the external world by the sense of sight. Or perhaps in the process of vision one who sees gets the picture from behind, while the examiner who looks into the eye would get it from before.

Before turning to more important things we add here a few words about the letters, or, more generally, the symbols, of ordinary printing. Professor Goudsmit has classified the capital letters with respect to symmetry.[3] The letter *F*, for example, being a real object, must be three dimensional. Its recognized pattern, if it appears on a flat surface, has two dimensions only, but the recognition of it by sight involves a third. Hence F, printed on thin paper and viewed from the back, may be ꟻ, ᖵ, Ⅎ or ᖶ, but never F. Accordingly, the picture of F, formed upon a flat opaque surface by a lens, and viewed from the front, will never be F. If it is ꟻ from the front it will be F from the back. Accordingly, the opaque projector contains one plane mirror, and the picture of the printed page, seen from in front, is legible. No doubt one could use a translucent screen, with light coming from behind, and then the mirror would not be required, or in general there would have to be an even number of mirrors, or none—probably a doubtful advantage. Conditions are different if a concave mirror is used for projection instead of lenses. Then the picture of Ⅎ, thrown on an opaque screen and seen from in front, is F. No plane mirror is required. Perhaps in former times girls were more prodigal of ink than they are today. It was a tragic wastefulness that enabled Jean Valjean to read by means of a mirror the reversed letter to Marius which Cosette had left upon her blotter. Or perhaps in those days ink evaporated more slowly.

[3] S. Goudsmit, "Symmetry of symbols," Nature **139**, 417 (1937).

The reflection of light by a plane mirror is familiar to everyone, but the consequences and implications of reflection are given too little thought. Suppose we have a pulley, 6 or 8 in. in diameter, and firmly fixed on a shaft a foot or so long that we can hold in our two hands before a plane mirror. If the axle is parallel to the mirror, and the wheel is turned, clockwise, for example, as seen from one side, then the image will be seen from the same side and will turn counterclockwise. If object and image were to be connected mechanically it would have to be by a crossed belt. The image of that part of the belt before the mirror will serve as the part behind, for the belt will cross at the mirror. But if the axle is held perpendicular to the mirror, object and image seem to turn the same way, and they may be connected by a straight shaft. However, if object and image were to be connected in the usual way by a flexible shaft, passing normally through the mirror, the image would turn properly for all positions of the object.

A shunt wound motor or generator may be represented as in Fig. 2, which enables the direction of turning to be predicted. The machine turns the same way as a motor and as a generator. If one thinks of a virtual image formed by a plane mirror behind the machine, it will appear that a motor like the image should actually turn as the image turns. I have never heard of machinery being run by the virtual image of a motor, but engineers are conservative.

We have been considering the motion of a virtual image when the object moves, but it is more important to follow the image when the

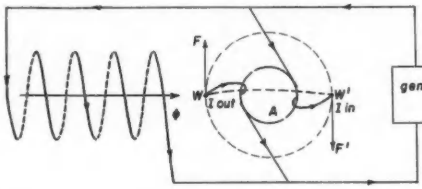

FIG. 2. Direction of turning, motors and generators.

mirror moves. Here the consequences of reflection appear in some unexpected effects. The action of the inverting prism, for example, seems at first, and even after considerable observation, quite mysterious to most people. For certain optical instruments a prism is made about as shown in Fig. 3. The acute angles are close to 45°. One may look directly through this prism endwise, as indicated by the traced ray. Then, if the prism is rotated about the long axis, the object will seem to rotate also about the line of sight, in the same direction as the prism rotates, but twice as fast; if the prism is turned 90°, the image turns 180°. In the Pulfrich interferometer such a prism is used to make the fringes appear vertical, no matter how the wedge-shaped space between the two plane plates happens to be placed. A right-angled prism will show the same effect if one will screen off about half of each face next to the right angle. To explain this double rotation we first make an experiment with a plane mirror.

Let the reader stand (or imagine that he stands) facing south, and let him hold a plane mirror in a north-south plane in his right hand. The mirror will be west of his head, and his image, though he may not see it, will face south also. Now let him swing the mirror around in front, and when it reaches the east-west plane the image faces north, so that the image has turned 180° while the mirror turned 90°. Continue the swing. When the mirror reaches the north-south plane, but now east of the observer, the image again faces south, and it has turned completely around (360°) while the mirror turned 180°. To make the mirror act more like the inverting prism, suppose the mirror held with its reflecting face toward a horizontal axis, so that the observer looks along the axis toward a familiar object, a house perhaps, and by turning his eyes a little can see the same object "in the mirror." When the mirror is vertical the mirror house is erect, though of course perverted, and a left-hand house if in reality right. Turn the axle and swing the mirror, and the image house turns twice as fast, so that when the mirror is below the axle and horizontal the mirror-house is wrong side up. This is exactly the effect of the inverting prism, but the two refractions permit the observer to look straight through instead of having to turn to one side. The effect of the prism can be produced by three reflections if three mirrors A, B and C

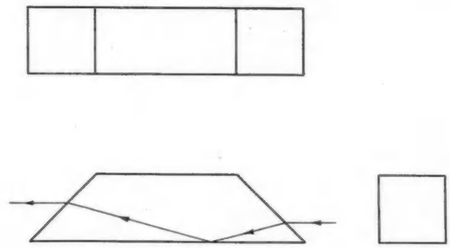

FIG. 3. The erecting prism.

are mounted perpendicular to a small board, more or less as indicated in Fig. 4. The reason for the double rotation is as follows.

In Fig. 5 let MN represent a plane mirror, and let AB be a line fixed in a real object, and $A'B'$ be the virtual image. If MN and AB turn together as one rigid body, clockwise, for example, in Fig. 5, then $A'B'$ turns clockwise also,

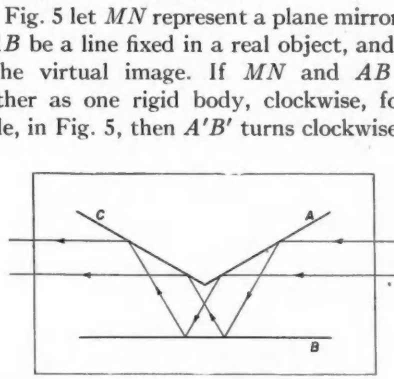

FIG. 4. Three mirrors replacing an erecting prism.

and just as much. But now let the mirror remain motionless and let AB turn counterclockwise about an axis through K. The image turns about K' and clockwise, but its angular velocity is the same as that of AB. Now let MN and the axis K turn as one body about O, so that AB moves, but let AB remain parallel to its original direction. The effect on the image is the same as if AB

rotated counterclockwise, since by remaining at rest it does move with respect to the moving mirror. Hence the image turns clockwise on account of the rotation of the moving mirror, and also clockwise as much more on account of the relative counterclockwise turning of the object.

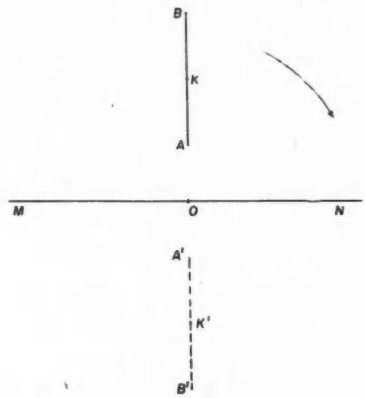

FIG. 5. Rotation of image caused by rotation of mirror.

Hence as the mirror alone turns the virtual image turns the same way but twice as much.

Now that we have shown in the most primitive way that the angular velocity of the image is twice that of the object, we can give a more elegant proof. In Fig. 6, MN is again the mirror, and AB is a line in the real object; $A'B'$ is the image of AB, and these lines, if continued, make

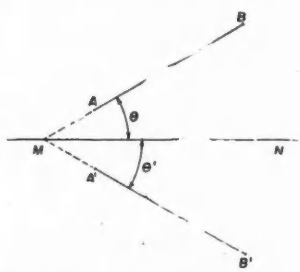

FIG. 6. The rotation of the image is double that of the mirror.

equal angles θ and θ' with the mirror. If now AB remains fixed, but the mirror turns clockwise, θ will increase by some amount $\Delta\theta$, and θ' will increase by the same amount. Hence $\theta+\theta'$ will increase by $2\Delta\theta$, and this means that the image

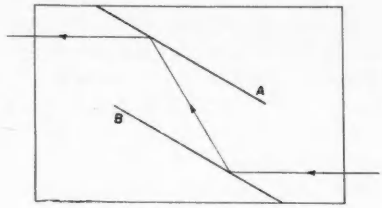

FIG. 7. Parallel mirrors.

turns twice as much as the mirror turns. It is clear that if the mirror rotates about some line in its plane, a line in the image parallel to that axis will not change its direction. What is said here refers to any line in the image in a plane perpendicular to the axis on which the mirror turns. It is frequently shown that a reflected ray of light turns through the angle 2θ if the mirror turns by θ, but the rotation of a line in the image is not often mentioned. We should have some elementary problems on this point, such as these.

(1) An observer faces a plane vertical mirror. The mirror rotates 10° on a vertical axis. What happens to the image? Give a numerical answer and prove it.

(2) Suppose one mounts two plane mirrors A and B, each perpendicular to a small board and parallel, as indicated in Fig. 7. What will happen to the image if one looks straight through, as indicated by the ray, and turns the board about the line of sight? One may use a single block of glass for this experiment, for example, a Fresnel rhomb if such a piece is available. Then there will be two refractions and two internal reflections.

(3) Suppose one mounts two plane mirrors on the table of a spectrometer, both vertical, and arranges the telescope and the collimator, and the two mirrors, so that he can set the image of the slit on the cross-wire, using light reflected once from each mirror. How much must the telescope be turned in order to follow the image if the table (with the mirrors) is turned 10°? What is the bearing of this experiment on the use of the pentagonal prisms in a range finder? Explain.

The action of the two 90° prisms in the familiar prism binocular is explained as follows. Let the reader face toward the south and hold before him a vertical plane mirror in the east-west plane. His own image is perverted, having the familiar scar on its left cheek instead of on its right, and faces north. Now let the mirror be turned 45° on a vertical axis, counterclockwise as seen from above. The image turns 90° and now faces west. Now a second mirror, vertical and in a north-south plane, receives the light reflected from the

first, and forms an image of the image. This second image is natural, not perverted, and faces east. The second mirror is turned 45° on a vertical axis, counterclockwise also. It now stands at 90° to the first mirror, and the final image is erect and natural and faces north. This is the effect of the horizontal prism in the binocular, the one with triangular faces horizontal and rectangular faces vertical. It is easily observed when two plane mirrors are accurately joined in a 90° prism, especially if it is a large one. But now we recall that the binocular consists, to begin with, of two astronomical telescopes. In such a telescope the objective forms a real image, rotated 180° on the axis of the instrument (and therefore inverted) but not perverted.

We have been speaking of the formation of a virtual image when a real object is before a plane mirror. But in the binocular the reflections occur between the objective lens and the real image. Hence that image becomes a virtual object with respect to the mirror, and the image is real. This situation is never mentioned in general physics textbooks, and not one student in a thousand will figure out alone that a plane mirror can ever produce a real image. However, as far as rotation is concerned, the real image of a virtual object formed by one reflection is changed in the same way that a virtual image of a real object is changed, and is perverted with respect to the virtual object, that is, in this case, with respect to the real image formed by the objective lens, which is not perverted. Two internal reflections by the first prism, accordingly, leave us with a real image, not perverted, but still inverted (because the original real image is rotated through 180° on the axis of the telescope) and facing the other way because rotated through 180° about a vertical axis with respect to the original real image, or with respect to the actual real object that is being observed. The second prism now has its triangular faces and longest rectangular face vertical. Without following through in detail we can see that the second prism also forms at each reflection a real image of a virtual object, the final real image having been rotated 180°, with respect to the real image caused by the first prism, on a horizontal axis across the telescope. Consequently the final real image is erect, natural (that is, not perverted with respect to the original object) and facing the same way as the object being observed.

To realize that the four reflections in the two prisms have done no more than to turn the first real image through 180° on the axis of the telescope, the reader may take a closed book in his hands, the front cover toward him, so that he reads the title in the ordinary way. Now let him turn the book 180° about a vertical axis, and then turn it 180° about a horizontal axis transverse to the line of sight. The effect of these two rotations is the same as the effect of one rotation of 180° upon the line of sight as axis. One rotation of 180° upon the line of sight is the effect of the two prisms. Besides this, however, since they twice reverse the beam of light, they bring the real image closer to the objective than it otherwise would be. This permits the binocular to be made shorter than an ordinary astronomical telescope of the same magnifying power. Moreover, the common field lens that gives the erect image in the so-called terrestrial telescope has the effect of requiring the tube to be longer. Finally, one notes that the eyepiece forms a virtual image of the final real image, erect and unperverted; hence the final virtual image observed by the beholder is like the object, except that it seems larger. This is not exactly the same as to say that it seems nearer.

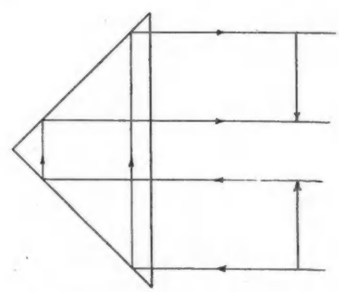

FIG. 8. Conventional diagram for prism binocular.

One might wish that there were a way of adding two half rotations, one about a vertical axis and one about a horizontal transverse axis, so that the sum would represent the resultant effect, a half rotation about a horizontal longitudinal axis. But if these angles of π rad each are represented by two vectors, each along the corresponding axis, the vector sum is not along the longitudinal axis.

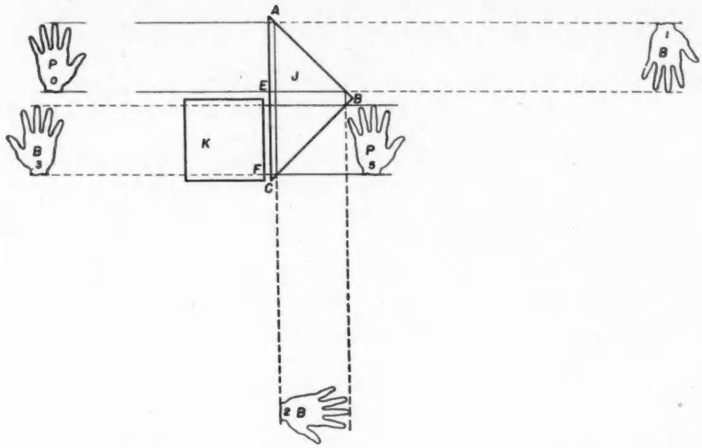

FIG. 9. Helpful diagram for prism binocular.

The vector product would lie along this axis, but it is somewhat too arbitrary to make the vector product of π and π equal π and the vector product of 2π and 2π equal 0.

Prism binoculars are referred to in many books, but I have not found the real image of a virtual object, as formed by a plane mirror, mentioned in any text, nor any correct account of the perverted image. Several elementary textbooks confuse perversion with the horizontal reversal from right to left that always accompanies inversion when the axis of the lens or mirror is horizontal. Many do not mention perversion at all. It should be made clear that only a three-dimensional figure, for which the term image should, it seems to me, be reserved, can be perverted. For, as has been said before, an actual right hand is a right hand, no matter how one looks at it. But the two-dimensional picture of a right hand (on a translucent screen) is right or left, depending on the side from which it is regarded. Prism binoculars are usually dismissed with a single diagram like Fig. 8. Here the arrows do not represent object and image, and the reader is never told what they do represent. It is a most extraordinary student who can get the correct idea from such a figure.

In order to be of any real help to a student trying for the first time to understand the prism binocular, some such diagram as Fig. 9 is necessary. Even this is over-simplified, because, of course, the images are formed by converging rays, and the bending of these rays at the air-glass surfaces is not shown in Fig. 9. In this figure the original object, a right hand, is O, and the successive images are numbered in order; $1B$ is the real image owing to the objective lens that forms the virtual object for the reflection at AB, the real image produced being $2B$. The image formed by the first reflection in the second prism (No. 4) is not shown, but it would be in front of the paper and toward the reader. All the images formed by reflection in the prisms are real images of virtual objects. The letters P and B indicate that the palm of the hand, or the back, respectively, is upward, the plane of the paper being the horizontal plane. The two prisms are J and K. No. 5 is the final real image forming the real object for the eyepiece, and since the eyepiece used as a simple magnifier forms an erect and unperverted image of its own object, the final virtual image is like the object in all respects, except that it subtends a larger angle at the eye of the observer than the actual object subtends. It is clear from Fig. 9 that the effect of the two reflections in prism J is to turn the first virtual object (here No. 1) through 180° about a vertical axis. This axis is perpendicular to the plane of the figure, and actually vertical when the binocular is held horizontal. The effect of the prism K is to rotate No. 3 about a horizontal axis, giving No. 5. This half-turn is no doubt the effect intended to be conveyed by the diagram of Fig. 8, but since Fig. 8 shows only one dimension where three are really necessary, it is utterly inadequate to convey the essential process.

The Logic of the Calendar[1]

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BECAUSE of the importance of chronological sequence in the task of relating and interpreting human experience, man has from remote antiquity felt the need of a naturally ordered time scale. Also, because of the extent and variety of man's activities—related as they are to his religion, philosophy and science; to his economics, politics and social orders; to his language, literature and folk lore; and, more particularly of late, to his business affairs, governmental and legal commitments, and industrial enterprises—and because the time scale itself must inevitably become an integral part of the record which this scale correlates, it is apparent that the problem of calendar construction is one of major proportions. It is not to be expected, therefore, that a solution of the problem adequate to the total requirements of an increasingly complex civilization should at first have been forthcoming.

Yet, from the manner in which the calendar is woven into the record of human history it follows that any attempt at revision is a task to be approached with the greatest caution and circumspection. Were the problem of calendar revision a matter of pure arithmetic only, we might be able to discover an entirely new solution or a simple revision that would better meet the requirements of modern society. But as has been pointed out, the calendar as we now have it is primarily a social institution—and a social institution that has stood more or less intact for 20 centuries cannot be set aside or essentially modified with impunity. We cannot "rub out and start over."

Yet we cannot let the wheels of progress be clogged with conventions of medieval origin. The alternative of calendar revision is, therefore, not so point-blank as the alternative proposed by Hamlet—"to be or not to be"—but leaves us rather with the questionable alternative of having to decide to be *and* not to be. This would seem to leave little from which to choose. Nevertheless, a careful analysis of the problem might be of value in enabling us to decide upon the necessary elements to be included, and the variations possible. In this manner, whether it is possible to reach a wholly satisfactory solution or not, at least we can decide what is possible and what is not possible in the urgent problem of calendar revision that now confronts us.

Even the earliest calendars were founded upon the orderly movements of the heavenly bodies. This implies careful and systematic observation.[2] Thus we seem justified in saying that the calendar represents one of the first attempts to apply in a large way the results of man's observation of natural phenomena to the needs of his daily life. At least this aspect of the calendar should commend itself to students of natural science.

INCOMMENSURABILITY OF THE FUNDAMENTAL UNITS OF THE CALENDAR[3]

Verily we live from day to day, and surely no one would suggest a time-reckoning system that does not include the solar day as a fundamental natural unit. Of course, we could stop here and the question of commensurability would not arise. Longer units could be arbitrarily selected. Intervals of say ten, a hundred and a thousand days, and so on, could be selected and given appropriate names. Or if we preferred the sexagesimal system, multiples of 12 and 360 could be used. The *Julian day number* extensively used in astronomy and in the preparation of interest

[1] Some of this material also appears in the author's article, "Problems of the calendar," J. Calendar Reform **10**, 175 (1940).

[2] As early as 2283 B.C. the Babylonians were making occasional astronomical observations. In 747 B.C., in the reign of King Nabonassar, the observations became continuous and form an unbroken series for over 360 years. This is the longest period of continuous and uninterrupted observations ever carried out, even if we include the records of modern astronomy. These observations were of great value in establishing the Babylonian calendar. See Breasted, *Ancient times* (Ginn, 1935).

[3] We are not overlooking the possibility of long-range time-reckoning devices based on purely physical phenomena, such as the decay of radioactivity, nor even of biological or other cyclic phenomena. But for the purposes of this article we are ignoring all time-reckoning systems not based upon the movements of the celestial bodies.

tables is an example, though multiples of the day are not used. To find the interval in days between any two dates for which the Julian day numbers are known, we merely have to subtract the smaller Julian day number from the larger. It is as simple as this.

But just as truly do we live year in and year out. Hath the poet not written:

> Perceiv'st thou not the process of the year,
> How the four seasons in four forms appear
> Resembling human life in ev'ry shape they wear?
> Dryden: *Of Pythagorean Philosophy.*

Surely no one would suggest a calendar that ignored the tropical year any more than we would suggest one that ignored the solar day.

This gives us two fundamental natural units, each indispensible to a calendar that is to measure human history in terms of two natural cycles which largely condition this history. Unfortunately, the durations of the tropical year and the solar day are incommensurate; that is to say, there is no integral number of mean solar days in a tropical year. The tropical year contains very approximately 365.2422 mean solar days. Obviously, a practical calendar that employs these two incommensurate natural units can only be perpetuated by patching it up from time to time by the deletion or addition of one of the units, or by the substitution for one of the natural units an arbitrary practical unit which approximates the natural unit as nearly as it may and which at the same time is made commensurate with the other of the two natural units; or by a combination of both devices. This procedure, called *intercalation*, is extensively used in the construction of practical calendars.

The *Julian calendar*, precursor of our current Gregorian calendar, was initiated by Julius Caesar with the assistance of the astronomer Sosigenes in the year 45 B.C.; its construction illustrates the procedure of intercalation. The Julian calendar year of 365.25 mean solar days was substituted for the tropical year of 365.2422 mean solar days, which latter served merely as the natural basis of the calendar. In practice, the quarter-day of the calendar was allowed to accumulate for three successive years and to the fourth year a full day (leap-day) was added to the calendar. This provided a four-year calendar cycle having three ordinary years of 365 days each and one leap year of 366 days, and provided a calendar year having an average length of 365.25 calendar days.

Two unsatisfactory results grew out of this apparently simple and "practical" solution of the difficulty. First, it established two actual calendar years of unequal length. Second, it left an uncompensated excess in length of 0.0078 day [=11 min, 14 sec] of the calendar over the tropical year. This excess was allowed to accumulate and by the year 1582 amounted to about 14 days. Thus the date of the spring equinox, which originally (45 B.C.) fell on the date March 25, had receded in the calendar to March 11. Because of the manner in which certain important church festivals, particularly the date of Easter, were related to the equinox the eccelesiastical calendar began getting out of hand.[4] Accordingly, in the year 1582 Pope Gregory XIII, with the advice of the astronomer Clavius, proposed a revision of the Julian calendar with a view both to restoring the date of the spring equinox to March 21 (the date it had at the time of the Council of Nicaea) and to avoiding a repetition of its seasonal displacement. The date of the equinox was restored by suppressing 10 days from the then current calendar. This was put into effect by proclaiming that October 5 of the year 1582 should be designated October 15 of the same year. Students of history will recall what came of this drastic order. The historians refer advisedly to the year 1582 as the "year of confusion." A glance at the calendar for the month of October, 1582 herewith reproduced will sufficiently indicate the utter confusion that resulted.

October, 1582

S	M	T	W	T	F	S
	1	2	3	4	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

Imagine, if possible, what would happen if a similar calendar for the month of October 1945 were instituted by executive order!

[4] The Council of Nicaea in 325 A.D. had "fixed" the date of Easter as the first Sunday after the fourteenth day of the new moon that falls on or after March 21. The moon referred to here is a fictitious moon that follows the lunar cycle.

With the equinox thus restored to its desired ecclesiastical date, it was still necessary to fix its position. This was done as follows. First it must be noted that the uncompensated accumulation of 0.0078 day amounts to one day in almost exactly 128 years. That is to say, to compensate accurately the Julian calendar one day must be deleted in 128 years, or 3 days in 384 years. Thus the deletion of 3 days in 4 centuries provides a close approximate compensation. This is what was done by the so-called century leap-year rule, which is the distinctive feature of the *Gregorian calendar*, namely, that every year divisible by 4 shall be a leap year except century years, which latter remain ordinary years unless evenly divisible by 400. This rule establishes the average length of the Gregorian calendar year as 365.241 896 days as compared to the Julian calendar year of 365.250 000 and the true tropical year of 365.242 197. In other words, the Gregorian rule reduces the difference between the average lengths of the calendar and tropical years to 25.92 sec; whereas the difference in the case of the Julian and tropical years was 674 sec. This rule would permit the accumulation of 1 day in about 3300 years instead of 1 day in 128 years as in the Julian calendar. This would seem quite sufficient. Yet, when Russia adopted a revised calendar in 1918, a new rule was formulated to reduce the difference between the calendar year and the tropical year still further. The Russian rule is: "Century years shall be leap years only in case their numbers when divided by nine give a remainder of two or six." This rule reduces the average difference between the calendar and tropical years to 3 sec, and permits an accumulation of 1 day in 29,000 years.

If this is not sufficient for the most meticulous students of intercalation we would suggest the rule proposed by Professor T. A. Bickerstaff, of the University of Mississippi, to wit: "Every year divisible by four shall be a leap year excepting years divisible by 2^7." The elegance of the form in which Professor Bickerstaff states his rule depends, we think, on the manner in which he employs the venerable calendrical number 7; for 2^7 is, of course, equal to 128, which is almost exactly the compensation required. Under this rule it would require about 90,000 years for the calendar year to outstrip the tropical year by one day. This establishes the calendar year at 365.242 185 days.[5] Thus we can say that so far as the problem of incommensurability arising out of the use of the mean solar day and the tropical year is concerned, there are available several simple rules of intercalation that can accomplish all that seems to be required in this particular.

SECONDARY ELEMENTS OF THE CALENDAR: THE SEVEN-DAY WEEK, AND THE TWELVE-MONTH YEAR

Devotees of the jig-saw puzzle will find special delight in attempting to fit the week and the month into the day-year pattern previously established. A perfect fit would require that we have an equal integral number of days in a week, an equal integral number of days in a month, and an equal integral number of months in a year; and that we have an equal integral number of weeks in a month; and, finally, that this integration apply equally well to the ordinary year and to the leap year.

But before entering this maze of multiples we must consider a few restrictions enforced by centuries of universal consent. There "must" be 7 days in the week, and there "must" be 12 months in the year. Much might be said in support of the inexorability of these two "musts," but since this is not permissible, let it suffice to say that the seven-day week[6] is the most firmly established of all institutions of human society. To delete the week from the calendar or to change its length is out of the question. Both expedients have been tried with utter failure.

[5] Students of physics might be interested to note that all this confusion growing out of the adoption of the Gregorian calendar came about by ignoring three points in the fourth decimal place. The seventh-power rule would have rendered the fifth decimal place secure. Nor was this all. The confusion did not end with the year 1582. The Russians continued the use of the Julian calendar until 1918; the Romanians until 1919; and the Greeks until 1923. Naturally more confusion resulted when these countries adopted the modified Gregorian, or *Eastern*, calendar, as it is called.

[6] "The seven-day week is of Semitic origin. Traces of it are to be found among Chaldean, Egyptian and even Greek records. Indeed, amongst many peoples the number seven seems to have been endowed with special significance. But it is among the Jews that the seven-day week was fully developed, and it is from them that its observance has spread over, and now so largely dominates, the civilized world."—A. Philip, *The calendar* (Cambridge Univ. Press, 1921), p. 29.

No less firmly established as a permanent fixture of calendar structure is the twelvefold division into months. Much might be said[7] of the useful properties and distinguished history of this "noble" number; but here again necessary limitations forbid. Numerically, 12 may be factored integrally into halves, thirds, quarters and sixths. The sheer usefulness of this facile number, according to Mr. Wells,[8] commended it to the culture even of Neolithic man. Its use in calendar structure comes down from the days of the Pharaohs. Its embodiment in human history necessitates its continuance.

Aside from the numerical aspect of the twelvefold division of the year, the name of each month has come to have an individual personality and distinctive connotations dear to the heart of every lover of Nature. This sentiment is feelingly expressed in the opening verses of Chaucer's *Prologue*:

> Whan that Aprillë with his shourës sootë
> The droghte of March hāth percëd to the rootë
> And bathëd every veyne in swich licour;
> Of which vertu engenderëd is the flour;

Of October, December, June and the others: what a wealth of memories each name engenders, not because of its etymological implications,[9] which are now entirely inappropriate, but because of the calendrical associations of each with the sequence of the seasons as established over a period of 2000 years of calendar usage. And, perceiving the four seasons as they are ordained in the heavens and established upon the earth, shall we not give them their rightful symmetry in the calendar? No one would seriously consider forsaking this rich heritage for the sake of simplified arithmetic. Yet something must be done with the arithmetic. Primarily the calendar is a document to live by, and secondarily a system of bookkeeping, but we all agree that the simpler the bookkeeping the more it contributes to the pleasure of living.

[7] See P. W. Wilson, *The romance of the calendar* (Norton, 1937), chap. 7.

[8] H. G. Wells, *Outline of history*, p. 63.

[9] October and December were originally the eighth and tenth months of the primitive Roman year, but are now, of course, the tenth and twelfth months of the Julian and Gregorian calendars. Who cares that June memorializes the name of a Roman general (Junius) when we think of it as the birth-month of the glorious summer sun?

EPOCH AND TIME-KEEPING

Time-keeping systems are required to answer two distinct questions: how long? when? The first calls for an interval such as a year, a day, an hour; the second calls for a date, or epoch, such as 12 noon, June 1, 1944. To provide the epoch there must be some agreement as to when the year and the day shall begin. In principle it is agreed that the *day* at any place shall begin when the mean sun crosses the antimeridian of the place, that is, at midnight. A given *date* first appears on the earth at midnight on the 180th meridian. Thus a given date also disappears from the earth on the 180th meridian at midnight on that meridian (or what is equivalent, at Greenwich noon).

Since the year contains an integral number of days it too must begin and end at midnight; but at what midnight? Obviously any midnight might be selected. In the Julian calendar the year was arbitrarily begun about a week after the winter solstice. The Jewish calendar begins the year at sunset near the autumn equinox.

It is interesting to note that because of the convention employed in the *civil time system*, successive days, and likewise successive years, do not begin simultaneously for peoples all over the globe. Thus in a sense there is no common *epoch* to the day or the calendar. The adoption of *universal* (*Greenwich*) *time* would not only synchronize timepieces all around the earth but would supply a common epoch to the day and year. The day and the year would then begin at the same instant for everyone on the earth. In these days of very rapid transit by airplane, the "hour of the day" as indicated by timepieces is even now losing its former intimate association with the "time of day" as indicated by the sun. In order to keep any sort of semblance between the two it became necessary years ago (in 1883 in the United States) to establish standard time zones of 1 hour each. Now a fast airplane traveling eastward or westward in latitude 45° could easily cross a time zone every two hours. To keep the clock and the sun somewhere in striking distance we must set the clock forward or backward 1 hour every 2 hours traveled. This hardly seems worth while just to keep up the illusion that the clock keeps sun-time. Every-

thing might be greatly simplified by expanding a single time zone to encircle the earth. Then it would not be necessary to change the clock, and at the same time we should have a good chance of being about as nearly in step with the sun as at present. Aside from establishing an absolute epoch for the day and the year, this convention would be exceedingly convenient in many other ways that will immediately occur to the reader.

EPOCHS, ERAS AND CYCLES

When we speak of the current calendar year as 1944, the question naturally arises: 1944 years since when, or since what? The point of beginning may be any unique event or occurrence, whether real or fictitious. It might be some event of human history; or it could be some naturally ordered event, such as an eclipse of the sun or moon, or some conjunction of the planets. The *Jewish era* counts from the assumed date of creation; the *Mohammedan era*, from the flight of the Prophet; the *Roman era*, from the founding of Rome; the *Christian era*, from the birth of Christ. The duration, or interval through which any count extends, establishes its era. The year 1944 establishes a count which is known as the Christian era.

Eras may be made up of a succession of cycles. Eras, since they are based upon unique events, are therefore distinguished from cycles in that eras are nonrecurring. There may be, as we shall presently find, cycles within cycles, for any common multiple of two or more different cycles may be used as a complex cycle. The science of chronology is largely concerned with epochs, cycles and eras.

SOME WELL-KNOWN CYCLES

The solar cycle.—This cycle establishes the recurrence of the same day of the week on the same day of the year. Since the ordinary year contains 52 weeks plus 1 day, a particular day of the year would come 1 day later in the week each succeeding year. Because of leap year this sequence is interrupted and the cycle will not repeat until $4 \times 7 = 28$ years. The solar cycle was instituted at the Council of Nicaea in 325 A.D., at which time the epoch of the cycle was fixed by setting the year 1 A.D. as the tenth year of the cycle. The position of any calendar year in the cycle can be found by adding 9 to the number of the year, dividing the sum by 28, and taking the remainder. Thus $(1944+9)/28 = 69+21$. Therefore, 21 is the number of the current year in the solar cycle.

The dominical letter.—To construct a calendar for a particular year it is necessary to know the day of the week on which a given date will fall. To accomplish this in general, it is expeditious to use a device known as the *dominical letter*, which designates the day of the month on which the first Sunday of the year falls, using the seven letters

A	*B*	*C*	*D*	*E*	*F*	*G*
1	2	3	4	5	6	7

for this purpose. Thus if the first Sunday falls on January 2, as it did in 1944, the dominical letter of this year is *B*, the second letter in the foregoing sequence.

To find the dominical letter of any year the following rule has been proposed: (*a*) divide the number of the year by 4, and disregard the remainder; (*b*) add the quotient to the number of the year; (*c*) for dates between 1800 and 1899 subtract 7 from the sum; from 1900 to 2099, subtract 8; (*d*) divide by 7 and note the remainder; (*e*) take the corresponding letter in the following table as the dominical letter:

1	2	3	4	5	6	7 or 0
G	*F*	*E*	*D*	*C*	*B*	*A*.

Thus, to find the dominical letter for the year 1945, we have (*a*) $1945/4 = 486$ (quotient) $+1$ (remainder), (*b*) $1945+486 = 2431$, (*c*) $2431-8=2423$, (*d*) $2423/7=346+$ a remainder of 1; (*e*) from the table the dominical letter for 1945 is found to be *G*. For leap year the rule is a little more complicated: there are two dominical letters, one to be used up to February 28 and another to be used for the rest of the year. The dominical letter is used in finding the date of Easter and is also the basis of many of the so-called perpetual calandars.

The lunar cycle of Meton.—This cycle establishes the recurrence of the new moon on the same date each year of a solar calendar. The lunar, or synodic, month equals very nearly $29\frac{1}{2}$ solar days, whereas the tropical year equals 365.2422 solar days. The Greek astronomer Meton (432 B.C.) pointed out that 19 solar years were very nearly equal in length to 235 lunar months. He thus established this period as the *lunar*, or *Metonic, cycle*. The epoch of the cycle is fixed by the new moon which falls on January 1. It was introduced into chronology about 530 A.D. and is useful in determining the date of Easter and on other occasions where lunar and solar phenomena must be correlated with respect to the calendar.

The *golden number* of a year denotes its position in the Metonic cycle. To find the golden number of a year, add 1 to the year and divide by 19. The remainder is the golden number of the year. For the current year, $(1944+1)/19 = 102+7$. Thus the golden number for 1944 is 7.

Epact, which is similar in significance to the golden number, specifies the age of the moon on January 1. The moon was new on December 27, 1943. It was five days old on January 1, 1944. The epact for the year 1944 is, therefore, 5. Epact is used in connection with the lunar calendar in a manner similar to the way the dominical letter is used in the solar calendar. The complete cycle of epacts covers 7000 years.

One of the simplest rules[10] for finding the *date of Easter* makes use of the golden number, the dominical letter and the epact of the year. The earliest date on which Easter can fall is March 22; the latest is April 25, as in 1943.

The saros.—This cycle establishes the period of recurrence of similarly circumstanced eclipses. Since eclipses are possible only when a new or a full moon occurs near a node of the moon's orbit, it follows that the recurrence will be determined by the commensurability of the lunar month with the *eclipse year*, or interval required for the sun to pass around the ecliptic with respect to one of the moon's nodes. Because of the recession of the moon's nodes, which travel backward along the ecliptic to meet the oncoming sun, the eclipse year is shorter (346.620 031 days) than the tropical year. It is found, however, that 19 eclipse years (6585.7806 days) very nearly equal 223 synodic months (6585.3211 days). Thus the major circumstance of a given eclipse will be very nearly reproduced after this period; 19 eclipse years are equal to 18 calendar years and 11 or 10 days, depending upon whether four or five leap years intervene. Thus the total solar eclipse of May 28, 1900, the path of which struck across the Southeastern states, was repeated under very similar circumstances on June 8, 1918, after an interval of 18 years, 11 days. The saros, though extremely important in astronomical calculations, must not be confused with the Metonic cycle, to which it bears certain similarities. The Metonic cycle is not used in eclipse calculations, nor the saros in calendar cycles.

The Dionysian period.—The solar cycle restores the day of the month to the same day of the week. The lunar cycle restores the new or the full moon to the same day of the month. By combining the two cycles we can restore the new or the full moon to the same day of the week and the same day of the month. The length of the cycle is, therefore, equal to $28 \times 19 = 532$ years. This cycle covers the period of recurrence of the dates of Easter.

The Roman indiction.—This is a 15-year cycle that was established by Emperor Constantine (312 A.D.). It has no astronomical basis, but is useful in establishing other cycles. The epoch of the cycle is fixed by taking the year 1 A.D. as the fourth year of the cycle. The position of any calendar year in the Roman indiction is found by adding 3 to the number of the year and dividing by 15. The remainder is the number of the indiction. For the current year, $(1944+3)/15 = 129+12$. Thus the Roman indiction for 1944 is 12.

The Julian period.—In the year 1582 Joseph Scaliger announced a comprehensive cycle which he called the *Julian period*. This cycle is the common multiple of the 19-year Metonic cycle, the 28-year solar cycle and the 15-year Roman indiction. Scaliger pointed out that these three latter cycles when carried backward had a common beginning in the year 4713 B.C. (Julian calendar). He thus fixed the date January 1, 4713 B.C. as the beginning of the comprehensive Julian period. The length of this period is $19 \times 28 \times 15 = 7980$ years, and hence the first Julian period will not be completed until the year 3267 A.D. To find the position of any year A.D. in the Julian period add 4713 to the number of the year. Thus the year 1944 is the 6657th year of the Julian period.

Using the Julian period as a basis, Scaliger set up his *Julian day number*, which assigns to each day of the calendar a number representing its numerical succession since the beginning of the Julian period. The Julian day number of January 1, 1944, is 2,431,091.

[10] Sky **3**, 28 (June 1939).

SOME OF THE BETTER KNOWN ERAS

The era of Nebonassar.[11]—This era was instituted by Nebonassar (Nabu-nazir), King of Babylon in 747 B.C. (Julian calendar). The first and historically the most interesting of all the eras, it was used extensively by Hipparchus (2nd Century B.C.) and Ptolemy for expressing the dates of celestial phenomena. Because of the large number of celestial events, particularly eclipses, that are recorded in the *Almagest* and elsewhere, the epoch of the era is firmly established. The year 2693 of the era of Nebonassar began April 24, 1944, Julian calendar. "The era of Nebonassar is an appropriate reminder of both the geographical region where modern astronomy had its source and the remoteness of the period of history at which its development began."[11]

The Jewish era.—This era is dated from the traditional date of creation, which is set at 3761 B.C. The year 5705 of the Jewish era began at sunset on September 17, 1944 (Gregorian calendar). Since the Jewish calendar is a lunisolar calendar running on the 19-year Metonic cycle, the year begins just after the new moon following the autumnal equinox. The date of the New Year may vary from September 5 to October 5. A year in the Christian era may be converted into the corresponding year of the Jewish era by adding 3761.

The Christian era.—This era was established by Dionysius Exiguus about 533 A.D. He assumed that Christ was born in the 28th year of the reign of Augustus. January 1, 1944, Julian calendar, corresponds to January 14, 1944, Gregorian calendar.

Mohammedan era.—The Mohammedan calendar, unlike the Jewish, is strictly lunar. The era begins with the flight of the Prophet, Friday, July 19, 622 A.D. It was first used by Calif Omar in 640. The year 1364 of the Mohammedan era begins at sunset on December 16, 1944, Gregorian calendar.

The Grecian era.—This era, also called the *era of Seleucidae*, dates from the reign of Seleucus Nicanor in 311 B.C. The year 2256 Grecian era began, according to the present-day usage of the Syrians, on September 1, 1944.

Era of Diocletian.—"The early Christians in the East dated from the persecution under Diocletian. Hence this is sometimes called the *era of the martyrs*."[12] The epoch is September 17, 284 A.D. The year 1661 of the era of Diocletion began on August 29, 1944, Julian calendar.

The Roman era.—This era is reckoned from the supposed date of the founding of Rome, which, according to Marcus

[11] E. W. Woolard, "The era of Nebonassar," Sky and Telescope **1**, 9 (Apr. 1942).

[12] Wm. H. Barton, Jr.

Terentius Varro, is set at 735 B.C. Dates in this era are marked A.U.C. (*anno urbis conditae*, meaning "from the building of the city" of Rome). The year 2697 A.U.C. began on January 1, 1944, Gregorian calendar.

The Byzantine era.—This era was adopted at Constantinople in the seventh century and is still used in the Eastern Chruch. It is founded on the assumed date of creation, 5508 years and 4 months before the beginning of the Christian era. The civil year of the era commences on September 1. The year 7453 of the era began September 1, 1944, Julian calendar.

The Japanese era.—The year 2604 of this era, being the 19th year of the period Showa, began on January 1, 1944, Gregorian calendar.

ANCIENT CALENDARS

Ancient calendars are legion. Several of them, however, continued in use over a sufficient period to establish important eras. Among these should be mentioned the Chinese calendar of Emperor Yao. The *Book of Records* (China) refers to a date in this calendar that corresponds to the date 2357 B.C., Julian calendar. This is probably the most ancient "calendar date" that can be identified with certainty. This calendar, which is still in force, has been used by a larger number of human beings over a longer period than any other calendar. It is a luni-solar calendar.

Other early calendars are the Babylonian, the Jewish and the Egyptian. In all of these we find the interminable struggle to reconcile numerically the year (sun), the month (moon) and the day (earth). Considering the unlimited possibilities of numerical meshwork which this problem holds out, and considering the numerical proclivities and genius of the Hindu, we might well imagine that he too would have had something to contribute on this subject. He had. And we might also suspect that when the Hindu mind attacked a problem of such fearful patterns, something marvelous would most likely result. It did. His grand conclusion of the whole matter is that the *Maha Yuga*, or *great age*, consists of 4,320,000 solar years, whereas the *Kalpa* equals a millennium of Maha Yugas, or 4,320,000,000 solar years. Thus, according to P. W. Wilson, "The Hindu calendars disclose majestic and awful vistas—past and future—along which we gaze with strained vision into ultimate origins and finalities." But as utilities these calendars leave much to be desired.

Another early calendar was devised by the Mayas. Largely preserved in the majestic ruins of this remarkable civilization in Guatemala and Yucatan, it is represented as a marvelously accurate and elaborate system of astronomical tables ingeniously related numerically. According to Clifford N. Anderson,[13] of Bell Telephone Laboratories, the remarkable calendar of the Mayas could identify any day in 65,000,000 years.

Historically, any calendar is largely the record of man's attempt to reconcile for purposes of reliable and convenient time reckoning the periods of the year, the month and the day.

1 tropical year	= 365.242 199 solar days
1 lunar (ordinary) month	= 29.530 588 solar days
1 tropical year	= 12.368 267 lunar months

These are the stubborn facts as ordained in the heavens. What can be done to reduce them to a simplified integral arithmetic of daily life? In the answer to this question lies the story of the calendar.

To illustrate the difficulty of finding a simple common multiple of these numbers we note that the interval of 128 years contains an integral number of days very exactly. Thus 365.242 199 days$\times$128 years equal 46,751.001 47 days. On the other hand, we recall that Meton in 460 B.C. discovered the near equality in duration of 19 years and 235 lunar months. The difference in the two intervals is only about 30 min. Yet when we examine the number of lunar months in the interval of 128 years we find this to equal 1583 plus the considerable fraction 0.13818 day, or 3 hours and 18 minutes, which is responsible for the complications of the various luni-solar calendars. This seems nevertheless about the best bargain we can make. Extending the interval to larger multiples of 128 years merely leads us farther and farther into the "majestic and awful vistas" already exploited by the Hindus.

A SUGGESTION FOR THE FUTURE

Having emphasized the difficulties of calendar construction encountered in the past, we might be expected to offer some suggestions regarding the future. We do not, of course, presume to

[13] Sky **3**, 8 (June 1939).

announce the perfect answer to this age-old problem. However, from the foregoing considerations several practical ideas emerge that may be worth mentioning. First, the serious difficulty of reconciling a simplified practical calendar for daily use with a long-range recurring cycle to cover the span of human history suggests that we adopt two plans of reckoning: one, an annual arrangement permanent in form and symmetrical in structure after the fashion of "The World Calendar" which preserves an arbitrary 12-fold monthly division, without reference to the lunar cycle, and the 7-day week; another, for long-range reckoning; for this the Julian period with its practical representation in the Julian day provides a perfectly linear scale against which the current annual calendars could be alined. For numerical convenience 46,751 days (128 years) along this scale could be set off as an integral division and given suitable notation if desired. January 1 of each annual calendar could be designated on this scale. This would then establish the epoch of the given calendar in the Julian period, and the notation of any day in this calendar could be easily found by adding the permanent day number in the annual calendar to the epoch. One or two additional days, as the case may be, must be added for the untabulated day of the ordinary and leap years.

This would provide a simple linear scale against which any and every date could be easily set in register. Since the epoch of the Julian period is established by two natural cycles—the lunar and the solar—in connection with an arbitrary fixed multiplier of 15 years (the Roman indiction), the epoch of the period is firmly established by observational phenomena.

THE LOGIC OF THE PROBLEM

The problem of the calendar is fundamentally one of correlating human history with the ordered movements of the heavenly bodies, and while these movements do not lend themselves to arithmetical representation with the integral simplicity demanded by modern society, yet we must acknowledge that an approximate reconciliation is possible by the devices of intercalation and tabulation. Furthermore, we must acknowledge that without the astronomical background on which all calendars, ancient and modern, have been founded, the chronology of human history would be utterly confused.

Whatever calendrical systems may be set up, the real problem of the calendar still remains the problem of agreement. No solution of the calendar problem can be regarded as entirely successful unless it leads to universal use by the peoples of the earth.

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Men of science must add a fifth freedom to their charter: "the freedom to exercise unquenchable curiosity." They must not be bribed by any false promises of political Messiahs who might interfere with their freedom to seek the truth; they must insist upon asking questions and getting their answers from Nature rather than from a Dr. Goebbels.—J. H. HILDEBRAND, quoted in *Nature* **152**, 306 (1943).

Evolution of the Three-Phase 60-Cycle Alternating-Current System

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SUPPLYING electricity as a public service is a comparatively young industry: scarcely three-quarters of a century has passed since it was initiated. Yet in this brief span it has evolved so rapidly that but few of the original elements are now discernible. Such is especially true of the networks used for transferring electric energy from generating station to distribution center. The story of the change in character of these networks—from the simple two-wire direct-current circuits of the late '70's to the complex interconnected three-phase alternating-current systems of the present—is a fascinating one.

RISE AND FALL OF DIRECT CURRENT

No doubt it will surprise many—especially those educated in this generation—to read that Edison was not the first to erect and operate central stations; that his famed three-wire direct-current system of transmission followed, not preceded, successful alternating-current systems; and that his system of illumination by crude incandescent lamps postdated, by nearly a decade, installation of the first commercially-operated system for electric illumination of public thoroughfares—by arc lights, a method already old at the time Edison opened the Pearl Street station in 1882. For Davy had discovered the electric arc in 1801, and in 1808 had fired public interest in arc lighting through demonstration of the brilliance of an electric arc struck between carbon rods maintained at the considerable voltage afforded by the (then just completed) great battery of the Royal Institution. But the carbon rods were quickly consumed; and the batteries of that and yet later days were both rapidly discharged and expensive to renew; whence commercial exploitation of arc lighting awaited, perforce, both development of improved types of arc lamps and the advent and gradual perfection of a more efficient and economical high voltage source—the steam-powered, electrically-excited, direct-current generator. Thus it was not until the late '70's that public arc lighting became economically feasible.

In the early '70's Gramme invented, patented, built, demonstrated and sold in quantity several types of direct-current generators for electroplating, powering motors and arc lighting. In 1876, at the Breguet manufactory for Gramme machines, Jablochkoff invented his famous "candle," an improved type of arc lamp. The Jablochkoff candle and the Gramme lighting generator were conjoined to furnish the first practical system for large scale electric lighting of public streets. Soon after, it was found that alternating current was better suited to the Jablochkoff candle; Gramme alternating-current generators for this purpose appeared in 1878. This was one of the first successful commercial applications of alternating current.

While the system just delineated was being developed, Charles F. Brush was developing his constant-current system. Following several years of experiment and an initial successful commercial installation in San Francisco, Brush introduced his direct-current system into New York City, commencing operation in lower Manhattan shortly before Christmas of 1880. Subsequent to these successes Brush systems appeared throughout the United States and in Europe; in the latter it competed with and eventually supplanted the Gramme-Jablochkoff system.

Coevally with Brush, numerous companies in the United States (notably, the Thomson-Houston Company) were developing and installing systems for arc lighting. In consequence, at the time Edison in this country and Swan in England produced the first successful systems for lighting by incandescent lamps, companies operating direct-current arc lighting systems were widespread, well established and prepared to offer formidable competition to the newcomers.

Once introduced, however, incandescent lighting quickly supplanted arc lighting as a general medium for electric illumination. For, though the intensity of the arc lamp was most advantageous for outdoor use, this very brilliance rendered it quite unsuited to interior use. Recognition of

this and other disadvantages—the danger attending high voltage circuits; the inconvenience of series operation; the need of frequent, even daily, replacement of the carbons—had spurred many to attempt invention of a satisfactory low voltage incandescent lamp. This end was achieved, independently and quite simultaneously, by Swan and by Edison. The latter, with characteristic energy and tenacity—"I made up my mind that the problem of the subdivision of the electric current could be solved and made commercial"—proceeded immediately to exploit his invention. Within little more than three years after grant of his first lamp patent he had designed, built, installed and put into operation (on September 4, 1882) a complete public power system, the first in America, but not, as is usually stated, the first *anywhere*. For several public systems (in that power was available to private consumers) were already in operation in England: at Godalming, in Surrey (1881); at Brighton (February 1882); and in London around Holburn Viaduct, where Edison and his English associates had started operation midway of 1882. None of these systems, however, was as elaborate as the Manhattan system with its component voltage-regulated network of 14 mi of armored underground conductor, fused links for overload protection, attached meters, paralleled loads and other rudiments of a modern underground distribution network.

One grave disadvantage, though, attended operation of this network. Range of service was severely limited in that to maintain the operating potential[1] (110 v) fairly constant over any considerable distance required conductors of such size that their cost was prohibitive. This restriction was somewhat eased by Edison's—and, independently, John Hopkinson's—invention of the three-wire system. Successful operation of the first installation utilizing this system—at Sunbury, Pennsylvania, in July, 1883—resulted in subsequent universal use of Edison distribution systems. But though the three-wire system approximately tripled the area that could economically be served by a central station, this increase was insufficient to meet the mounting demands upon the direct-current systems. Various direct-current schemes to circumvent this areal limitation were proposed and instituted by prominent electrical engineers of the time: Brush, Weston, Thomson and others. These men recognized that the key to the problem was to effect some method of transmitting power at high voltage and low current (thus enabling use of conductors of relatively small cross-sectional area) to the load points, where it was to be subdivided (to use a term of that day) and distributed at low voltage to the electric devices utilizing it. But the schemes they evolved were, in an electrical sense, structurally unsound, and attained only limited use; they were abandoned when areal and other operating difficulties were resolved through introduction of alternating-current systems in the late '80's.

The germ of the alternating-current system of distribution is to be discerned in an 1878 patent issued to J. B. Fuller for use of an alternator and connected induction coil for electrical distribution. Seemingly, however, Fuller proceeded no further with his idea; so that it was not until some years later that actual, practical systems were inaugurated: in England in 1884 by the firm of Messrs. Gaulard and Gibbs; in Hungary in 1885 by Ganz and Company, utilizing equipment designed by Zipernowski, Deri and Blathy, engineers in their service. The alternating-current system devised by this trio is essentially identical with that used today. Contrariwise, the series scheme used by Gaulard and Gibbs was soon superseded; but though their work was of no permanent value, electrical engineering is indebted to them for the attention they drew to the possibilities of alternating-current service.

As one means of advertising their system, Gaulard and Gibbs exhibited at the Turin exposition of 1884 a 25-mi transmission line erected between Turin and Lanzo and illuminated at both ends and at intermediate points by power

[1] The operating voltages on early direct-current systems and the distribution voltages on early alternating-current systems ranged from 100 to 125 v on two-wire systems. The range was fixed by the characteristics of the incandescent lamps of that time; but choice of the nominal system voltage was the designer's. Thus, 100 v, quite suited to the Swan lamp, was a favorite of English designers. The Edison lamp, however, was suited to a somewhat higher voltage: hence the early 110-, 115-, 120-, and 125-v Edison systems of this country. Development of the "high" voltage incandescent lamp eventuated in England in standardization on 230 v for domestic supply. In the United States, however, use of the original values of 220/110 and 230/115 (and, rarely, 240/120) v for domestic supply has persisted.

transmitted according to their system. In some fashion George Westinghouse became acquainted with the work of Gaulard and Gibbs and, always on the lookout for new ventures, first optioned (1885), then purchased (1886) the American rights to their system. For experimental purposes the Siemens alternating-current generator and the induction coils comprising this system were imported from England. These were turned over to William Stanley, a brilliant young engineer who had associated himself with Westinghouse early in 1884.

In 1883 Stanley had become interested in the Brush distribution system previously mentioned; had recognized the essential defect in it and other proposed systems—the inadequateness of the methods used for voltage regulation of the individual loads; had hit upon the idea of self-regulation through use of alternating-current; had worked on it after joining the Westinghouse organization. At the time Westinghouse proposed investigating the Gaulard and Gibbs systems, Stanley had worked out a scheme involving generation and transmission of alternating-current power at high voltage; reduction of voltage at the load points through use of paralleled step-down transformers with closed iron core, high voltage primary coil, and low voltage secondary coil; and distribution to loads paralleled across the low voltage coils. To work out the physical details Stanley went to Great Barrington, Massachusetts, where he equipped a laboratory and installed a single-phase distribution system comprising steam power plant, Siemens alternator, 4000-ft high voltage line and a half-dozen paralleled step-down transformers feeding incandescent lamps. In the Barrington laboratory Stanley also tested (1886) the use of transformers in step-up, step-down concatenation, the method now in general use. Since these Barrington experiments and additional experiments near Pittsburgh proved successful, the Westinghouse Electric Company (which had been formed early in 1886) undertook sale of the system.

In November 1886 the first commercial plant started operation in Buffalo. Its success resulted in numerous orders for alternating-current installations. To fill these orders the Westinghouse Company undertook to design, to develop equipment for and to install alternating-current systems throughout the country. This procedure brought them into direct conflict with the Edison Electric Light Company, at that time in virtual control of the incandescent lighting business. At heavy cost licensees of the Edison Company had installed extended low voltage direct-current distribution systems in the major cities; it now appeared that these investments were to be jeopardized through forced competition with a new and technically superior system. A bitter struggle ensued.

The supporters of direct current endeavored by legislative action and by discrediting remarks printed in the newspapers, trade journals and other periodicals of the day to undermine confidence in and to prevent the spread of alternating current; the Westinghouse Company countered devastatingly by publishing accounts of new and successful installations. By 1890 it was evident that alternating current could not be headed off; indeed, it was conceded that in many respects it was superior to direct current. In 1892 the Edison Electric Light Company and most of the other important electrical manufacturers not associated with Westinghouse were merged in the General Electric Company; it and the Westinghouse Company pooled their alternating-current patents; as alternating-current motors and other apparatus were developed, the Edison distributing companies likewise adopted alternating current; from these alternating-current systems rotating synchronous converters fed those direct-current networks already installed whereof the magnitude of invested capital was too great to permit abandonment; gradually even these were absorbed. Today these networks exist only as annoying remnants in a few large cities, and generated direct-current power, for the most part utilized for railway purposes, accounts for less than 1 percent of the total generation.

THE RISE OF ALTERNATING CURRENT

Alternating-current systems of today are preeminently three-phase 60-cycle high voltage systems. But unlike the Goddess Minerva this conjunction did not spring, full-formed, from the corporate minds that sponsored the polyphase system. Indeed, the earlier stages of the develop-

ment of the alternating-current system were marked by what have been termed—somewhat romantically—the "battle of the phases" and the "battle of the frequencies." Conflicts ranged (i) between the proponents of two-phase and of three-phase systems and (ii) among those who at one time or another advocated $133\frac{1}{3}$, 125, $83\frac{1}{3}$, $66\frac{2}{3}$, 60, 50, 40, 30 and 25 cycles as a standard frequency. Interestingly, each of these conflicts was eventually resolved by development in the art itself, rather than by the corporate strength supporting the winner.

When in 1886 the Westinghouse Company introduced the alternating-current system into America—European systems had been in operation for some years—alternating-current motors comparable in performance to available direct-current motors were not in existence. In consequence, the earliest alternating-current installations were solely for lighting purposes. At the time nothing was known of analytic transformer design; empirical consideration suggested that a comparatively high frequency was best suited to distribution transformer operation. Accordingly, the Westinghouse Company adopted 16,000 alternations per minute (as frequency was then expressed) as being sufficiently high for satisfactory transformer operation, yet easily afforded by the small single-phase 2000-rev/min, 8-pole belted generator they were then furnishing. For somewhat similar reasons their major competitor, the Thomson-Houston Company, standardized on 15,000 alternations per minute. Thus, considerations regarding apparatus determined choice of the first standard frequencies: $133\frac{1}{3}$ and 125 cycles.

As systems enlarged, it became essential to market generators of much greater capacity; accordingly, around 1890 the use of low speed direct-coupled engine-type generators was introduced from Europe. Such low speed machines, if operated at the frequencies then in use, would have required a prohibitive number of poles (200 for an 80-rev/min, $133\frac{1}{3}$-cycle generator). The Westinghouse Company, after careful study of the variables involved, selected 60 cycles, a compromise between higher values deemed better suited to transformer operation and lower values actually somewhat better suited to generator design. Between 1890 and 1893 a number of 60-cycle, single-phase systems were installed. As were the $133\frac{1}{3}$-cycle, single-phase systems before them, these early 60-cycle installations were devoted primarily to lighting; satisfactory commercial induction motors were not as yet on the market.

For though Tesla had announced his invention of the polyphase induction motor in May 1888, it required nearly a half-decade of expensive experiment for Westinghouse (which held the Tesla patents) and others to develop motors possessing satisfactory mechanical and electrical characteristics and operable at frequencies which could also be used for commercial circuits supplying lighting loads and the like. Such motors having been developed by the Westinghouse Company in 1892–93, the question of a suitable supply system arose. For various reasons—among the more important of which were that two-phase systems were less expensive than three-phase; that long distance transmission line operation with its lower cost for the three-phase system was then not an important factor in central-station operation concerned primarily with single-phase lighting circuits; and that some of the early forms of generator windings and the original form of the Tesla induction motor were more readily suited to two-phase than to three-phase work—the Westinghouse Company adopted and marketed the two-phase system.

On the other hand, the Thomson-Houston Company, faced with the necessity of avoiding infringement of the Tesla patents and already very active in pioneering long distance, high voltage alternating-current transmission lines, took up the three-phase system. By 1893 the General Electric Company (formed of the Thomson-Houston Company and others) had installed a number of three-phase systems, one of the more impressive of which consisted of a 10,000-v, 23-mi line from Mill Creek to Riverside, California. This system is often cited as the first commercial three-phase installation in the United States.

In the same year (1893) the Westinghouse Company managed an impressive display at the Chicago Columbian Exposition, an exhibit of induction motors, synchronous motors and two rotary converters, which conjoined with display

of their installation for furnishing electric lighting of the building and grounds to excite considerable admiration. This electrical triumph was a potent factor in the decision of the Cataract Construction Company, the promotion company for the Niagara Falls hydroelectric project, to award to Westinghouse the contract for the generators (and associated electric equipment) comprising the initial installation at the Falls, these machines to be built according to designs furnished by engineers of the promoting company.

Dr. George Forbes had designed for them a 250-rev/min, 8-pole (hence $16\frac{2}{3}$ cycle) generator comprising a central fixed armature surrounded by a rotating field structure. After examination of this design the Westinghouse engineers suggested a 250-rev/min, 16-pole (hence $33\frac{1}{3}$ cycle) machine, arguing that this frequency was much better suited to rotary converter operation (an important consideration in that among the chief customers were to be several companies requiring large blocks of direct-current power), would afford a greater range of speeds for induction motor operation, and would do away with the quite intolerable winking of incandescent lamps on $16\frac{2}{3}$ cycles. As a compromise, 250-rev/min, 12-pole, 25-cycle machines were adopted. In consequence of the publicity attending choice of this frequency for the Niagara project, in that it proved to be ideally suited to the low speed, engine-type generators then coming into use, as lights did not wink too noticeably, and as it was quite advantageous to early designs of synchronous converters, 25–cycles was shortly adopted as a standard low frequency.

While in the fashion just described, 60 and 25 cycles thus came into use as standard frequencies, they were not without competition. The Westinghouse Company, desirous of standardizing on 60 and 30 cycles, in some instances advocated the use of the latter in place of 25 cycles; at one time the General Electric Company endeavored to introduce 40 cycles as a substitute for both 60 and 25 cycles; and both $66\frac{2}{3}$ and 50 cycles found sponsors. But except for 50 cycles, still retained to some extent (chiefly in California), these frequencies were eventually dropped.

The battle of frequencies, therefore, finally narrowed down to the two standards of 25 and 60 cycles, two being accepted because it appeared that between them they covered fields of service so widely different that neither could be expected to cover all alone. Thus it appeared that 25 cycles was most suitable for low speed engine-type generators, for low speed induction motors, for synchronous converters as they then existed, and prospectively, for long distance transmission of power (because of better voltage regulation stemming from the smaller reactance drop). On the other hand, 60 cycles was much better suited to lighting, both arc and incandescent, and in reason adoptable for other than very low speed induction motors. Because of its more general advantages, 25-cycle systems gradually gained ascendance over 60-cycle systems and for a time threatened to encompass most new installations other than those where lighting directly from the alternating-current system was involved.

But as improved designs of 60-cycle induction motors enabled the hitherto superior performance of 25-cycle machines to be combined with the more numerous speeds afforded by 60-cycle machines; as hunting, commutation and other problems besetting 60-cycle synchronous converter operation were resolved; as gradual development of the turbo-generator made clear that the higher speeds afforded by 60 cycles offered attractive opportunities for improved designs; as the introduction of the regulating synchronous condenser aided in overcoming the voltage-regulation difficulties attending 60-cycle long distance transmission; and as yet other advances in 60-cycle apparatus were effected, the 60-cycle system came back rapidly.

By the end of the 'teens it was apparent that 60 cycles was in the fore. And as many years before three-phase systems had become standardized both for long distance transmission and for local distribution, it was clear that 60-cycle three-phase networks were to be the networks of the future. This promise was shortly fulfilled as in the '20's—the heyday of private construction—the '30's—the decade of the great Federal projects—and the present '40's—the country was overspread with new 60-cycle, three-phase systems.

Electric Units in Elementary Physics

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THE mks system of electric and magnetic units is being more and more widely used every year by physicists and electrical engineers. As far as physicists are concerned, the adoption of this system has unquestionably been accelerated by the excellent report [1] made in 1938 by a committee of the American Association of Physics Teachers, of which W. H. Michener was chairman. In order to determine the extent to which the mks system has been used in the newer elementary textbooks, a survey has been made of all textbooks in beginning college physics which have been published or revised since 1939. Of the 17 textbooks included in this survey, 12 do not use the system, although most of these mention it briefly. One additional textbook devotes about three pages to a discussion of the system, but the presentation is such that students are obviously not expected to use the system. Two textbooks use the system almost on a par with the cgs system, and two textbooks use it exclusively.

With regard to the question of rationalization when mks units are used, the four elementary textbooks which now use the mks system appear to be evenly divided. Two of them use the rationalized system and two use the unrationalized system, as far as the factor of 4π is concerned. The committee report previously referred to[1] favors the rationalized form of the mks system and stresses the importance of standardizing on one form.

It is a matter of opinion as to whether the progress that has been made in adopting the mks system is as rapid as could be desired. There may even be some who regret that any progress has been made in this direction. Regardless of personal preferences, however, the facts are that the mks system has gained a place in physics and electrical engineering which demands recognition, while at the same time we cannot yet dispense entirely with the other systems. It is the purpose of this paper to consider two objections which have probably retarded the adoption of the mks system and to point out how these objections are overcome by the application of ideas which have already been advanced by others. The first objection has to do with the choice of fundamental units; and the second, with the fact that the mks system adds to the multiplicity of units already existing.

The first of these objections will occur only to those physicists who are convinced that charge is the most fundamental of all the electric quantities. Such physicists may dislike the mks system because they have encountered it in presentations where electric current is considered to be more fundamental than charge. Allowing for personal preferences, the point to be made in this connection is that the mks system does not restrict the choice of the order in which electric quantities are introduced in the development of electric theory. Thus, although the coulomb is not the mks unit which was originally chosen as the primary standard, the subject of electricity may still be introduced by taking electric charge as the basic electric quantity. The coulomb can be introduced merely as a unit of charge for which the size has been arbitrarily chosen, and the student can learn later without serious shock that it must have required a little forethought to choose this unit so that the permeability of empty space turns out to have a particular value. The situation may be described in general for any system of units by saying that the quantity which is chosen as fundamental in defining the electric quantities need not be the same as the one which is taken as fundamental in choosing the arbitrary sizes of the units.

That the process of defining quantities is independent of the process of choosing the size of units has been recognized by Bridgman.[2] His principle of the absolute significance of relative magnitude states that the ratio of two quantities of the same kind must be independent of the

[1] Am. J. Phys. (Am. Phys. T.) **6**, 144–151 (1938).

[2] Bridgman, *Dimensional analysis* (Yale Univ. Press, 1931), p. 20.

units employed. This is another way of saying that quantities must be defined in a way that will hold for any choice of unit sizes. Rusk[3] has pointed out that priority in any system of units does not mean priority in experience. In a review of a book on the mks system, Pugh[4] criticizes the authors for having linked the dimensions of physical quantities too closely with standards. From a practical point of view, as long as the elementary student understands the interrelationships among the various units, he need not be concerned greatly over the division between fundamental and derived units. In fact, there is often an advantage to be gained in letting our choice depend on the problem at hand. For instance, in mechanics we commonly express work in dyne centimeters, which is equivalent to using the dyne as a fundamental unit.

The objection that the mks system adds to the multiplicity of units which already exists is based on fact at present. It is to be hoped that the exclusive adoption of the mks system in elementary textbooks will eventually reduce the number of systems of units, but the day is still far off when everyone can agree on any one system to be used in all situations. Also, some of the units previously used are so firmly fixed in the literature that they cannot be quickly discarded. The probable continued existence of a multiplicity of units is itself one of the chief reasons for adopting the mks system. This system encourages the use of a simple method for handling all units in electric problems. This is the well-known "substitution" method, in which the names of units are treated as algebraic quantities in physical equations. The processes of multiplication and division are applied to the units, and the units are never discarded except by cancellation. Desired transformations of units may be accomplished simply by appropriate substitutions according to the ordinary rules of algebra. This method is now widely used for problems in mechanics, and could be used in problems in electricity with either cgs or mks units. Its use with the cgs system, however, has been found so inconvenient that textbooks usually incline to the use of pure numbers in equations which involve both electric and magnetic quantities, even after having strongly stressed the importance of carrying units through mechanical problems. This inconvenience appears to result from a number of factors, including the difference in the dimensions of the electrostatic and the electromagnetic units of charge. The mks system avoids these difficulties, and the writer has found it conveniently adaptable to a consistent handling of units by the substitution method.

The afore-mentioned substitution method of handling units has been advocated by Webster[5] and by Bridgman.[6] It is of interest to note that both of these writers recognize the simplicity and effectiveness of this method, although their viewpoints otherwise are quite divergent. Webster considers that the symbols in physical equations represent concrete quantities, and advocates the extension of the definition of multiplication so that it applies to concrete things as well as to abstract numbers. Bridgman insists that the symbols in physical equations represent pure numbers only, and that any process of substitution, multiplication or division applied to the name of a unit is merely a part of a convenient shorthand method of transforming units. V. F. Lenzen[7] has suggested a compromise which may satisfy both those who use symbols to stand for physical quantities and those who hold to the idea that they must represent pure numbers. He recognizes a way of defining a physical quantity which is abstract enough so that the processes of multiplication and division can be applied to it, and concrete enough so that it will have a definite value regardless of the units which are used. Whatever the philosophic viewpoint may be, the agreement that both a number and a unit are to be substituted for a letter symbol in equations means that the equations can serve as general equations which hold regardless of the units in which the quantities are expressed. For any given quantity, if the unit is larger, the number of units will be less, and hence the product which is to be substituted for a letter symbol will have a value that is independent of the choice of the unit.

[3] Rusk, Am. J. Phys. **9**, 57 (1941).
[4] Pugh, Am. J. Phys. **9**, 61 (1941).

[5] Webster, Science **46**, 187 (1917); Am. J. Phys. (Am. Phys. T.) **2**, 149 (1943).
[6] Reference 2, chap. 3.
[7] V. F. Lenzen, Am. J. Phys. **8**, 335–8 (1940).

To illustrate the effectiveness of this method, let us compute the force in newtons on a wire 2 m long carrying a current of 3 amp in a magnetic field of intensity 20 oersteds. Since it is easy to remember that $\mu_0 = 1$ gauss/oersted, let us write $B = 20$ gauss. Using the equation $F = BIL$, we write

$$F = (20 \text{ gauss}) \times (3 \text{ amp}) \times (2 \text{ m}), \tag{1}$$

with total disregard for any division of units into systems. Since 1 gauss = 1 dyne/abamp cm, we can substitute in Eq. (1) to get

$$F = 20 \text{ (dyne/abamp cm)} \times (3 \text{ amp}) \times (2 \text{ m}). \tag{2}$$

Then, using 1 abamp = 10 amp, 1 newton = 10^5 dynes and 1 m = 100 cm, we can substitute again to get

$$F = 20 \frac{(\text{newton}/10^5)}{(10 \text{ amp})(\text{m}/100)} \times (3 \text{ amp}) \times (2 \text{ m}), \tag{3}$$

or

$$F = (20 \times 3 \times 2 \times 100/10^5 \times 10) \text{ newtons}. \tag{4}$$

Instructors as well as students are pleased with the simple certainty of this method of handling units, and there is no necessity of remembering just which units must be used together in each equation. The derivation of routine numerical equations such as appear in handbooks for any given combination of units is easily carried out by the method. For example, it is obvious from the preceding example that we can write $F = 10^{-4} HIL$, where F is the *number* of newtons, H the *number* of oersteds, I the *number* of amperes, and L the *number* of meters. In this connection, many elementary textbooks are to be criticized for shifting from the use of general equations involving physical quantities in mechanics to the use of limited equations involving pure numbers in electricity, without any mention of the change in viewpoint.

It should be noted that this method of carrying units through equations also serves a useful purpose in continually reminding the student of the relationships among the quantities he is using. For example, every time a student obtains a joule per coulomb from his calculation for potential, it serves to fix his concept of a volt. In this way the student naturally gains the benefits he would derive from a study of dimensional formulas, without the introduction of additional equations with awkward square brackets, and without the introduction of terms and ideas that are either obscure or controversial to most physics teachers.

Doppler Effect—A Lecture Demonstration

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NEARLY every instructor of elementary physics appreciates the difficulty of effectively demonstrating the Doppler effect to a group of students. Several attempts have been made, notably the well-known revolving whistle, and a large tuning fork moved near a wall.[1] A better method[2] was introduced recently in which two loudspeakers each emitted the same note. One was stationary while the other swung as a pendulum, thus producing beats. The last two methods produce intensity beats, that is, beats in which the intensity varies with time. These are, in general, effective, except with some students who seem to have difficulty detecting the variation of intensity.

If, instead, two sources of different frequencies are used, they will produce beats even though stationary. Further, if these two are sufficiently different in frequency, the beat note will be of audiofrequency, and can be made audible by proper means. Now any motion of one speaker will result in a Doppler change of its apparent frequency which in turn will produce a change in the frequency and pitch of the audible beat note. This change in pitch is apparent even to the student who has difficulty in recognizing the intensity beats.

* The assertions contained herein are the private ones of the writer and are not to be construed as official or reflecting the views of the Navy Department or the Naval Service at large.

[1] J. Zeleny, Am. J. Phys. **9**, 174 (1941).

[2] F. E. Fox, Am. J. Phys. **12**, 228 (1944).

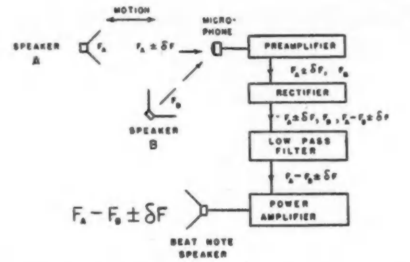

FIG. 1. Schematic diagram of apparatus for demonstrating the Doppler effect.

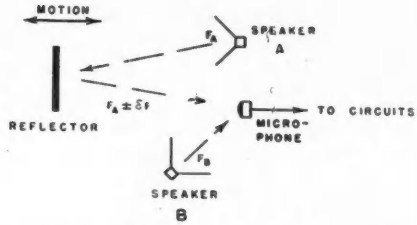

FIG. 2. Modified apparatus for showing enhanced Doppler effect.

Figure 1 shows schematically the apparatus for such a demonstration. The microphone receives sounds of two different frequencies. The output of the microphone is amplified, then rectified (following the procedure that is standard in superheterodyne radio receivers), so that the beat frequency actually exists in the circuit. The high frequencies F_a and F_b are filtered out, and only the beat frequency is amplified and made audible to the audience.

The student can see the two speakers A and B, and hear their individual tones. The electric circuits are best contained in one box, and only their general operation should be mentioned to students who have not yet had any electricity. The beat note can be shown to be such by varying the frequency of either A or B, and noting the resultant variation of the beat note. This also anticipates the reverse argument that a variation in the beat note frequency is held to indicate a variation in either A or B.

Now if speaker A is moved toward the microphone, a very definite rise in the frequency of the beat note is observed, and it is concluded that this indicates an apparent rise in the frequency of the sound emitted by A.[3]

Recently, J. O. Perrine[4] has rather exhaustively discussed various possible instances of the Doppler effect; he points out that if both source and receiver are stationary and the emitted sound is reflected back to the receiver by a moving reflector, the Doppler effect is increased nearly by a factor two. Figure 2 shows a modification utilizing this "enhanced Doppler" to emphasize further the effect and make it more apparent.

The author has used this modification in lectures to students, and has found it to be effective. When F_a is 5000 vib/sec, F_b is 4800 vib/sec, and the reflector is moved at 5 ft/sec, or 3.4 mi/hr (a walk), the frequency change is 45 vib/sec. With the undisturbed beat note at 200 vib/sec, 45 vib/sec represents a change of about 3.5 half-tones, which is easily perceptible even to the untrained ear. The amplified 200-vib/sec note is easily heard in the lecture room, distinct from the high-frequency notes of speakers A and B.

[3] The demonstrator is warned that for a simple demonstration F_a must exceed F_b; otherwise a rise in F_a will *reduce* the beat-note frequency, an event that can be very confusing to a first-year student.

[4] J. O. Perrine, Am. J. Phys. **12**, 23 (1944).

The scientist should be a man willing to listen to every suggestion, but determined to judge for himself. He should not be biased by appearances; have no favorite hypothesis; be of no school; in doctrine have no master. He should not be a respecter of persons, but of things. Truth should be his primary object. If to these qualities be added industry, he may indeed hope to walk within the veil of the temple of nature.
—MICHAEL FARADAY.

A Lens for a Miniature Camera

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Many of us who use small cameras such as the Leica, whether for technical purposes or for pleasure, have looked longingly at the variety of lenses available. We should like to have at least one lens of long focal length, but for various reasons, including the well-known one, have not acquired such a lens. The Leica camera, for example, is ordinarily equipped with a lens of focal length 2 in., with maximum relative aperture of $f/3.5$ or $f/2$. These lenses are the best general-purpose lenses, but for many uses, such as photographing at the zoo, the images are inconveniently small, owing to the distance of the subject. A lens of focal length 5 in., on the other hand, will yield images whose linear dimensions are 2.5 times those of the lens of shorter focal length when used at the same distance, with a correspondingly more satisfactory result. Before the war, such a Leica lens (f, $5\frac{3}{8}$ in.; relative aperture, $f/4.5$) cost $132, an expenditure that would ordinarily be made only if the lens were to be much used. Since I have found a solution for this problem, I take this method of presenting it to others who may be interested.

A photographic objective to be used with a camera yielding a very small negative, which is subsequently to be enormously enlarged by projection, should have certain characteristics. The field should be flat, and should be free from astigmatism, coma and distortion. The image of a multicolored object should be sharp, and its edges should not look like rainbows; that is, the lens should be chromatically corrected. As photographic objectives are constructed today, it is impossible to make a lens, even a costly one, simultaneously free from all of these defects. The matter therefore must be handled in a practical manner: There must be a certain amount of adjustment during design in allowing for the various faults just mentioned, the final design depending largely upon the use to which the lens is to be put.

As an example, consider the Leica objective known as the "Summar." This lens is a "fast" general-purpose lens, focal length 2 in., maximum relative aperture $f/2$. The makers state that it yields the best definition when used at relative aperture $f/6.3$. If this lens is used for general work, such as outdoor photography and portraiture, the results are excellent. But if the lens is used wide open to photograph a star field, coma will be very evident; and if it is used at the best aperture $f/6.3$ to copy a map, the corners will be out of focus when the center is sharp. In each of these instances the lens has been used in a way for which it was not designed. In design, certain features have been sacrificed in order to get a general-purpose lens having a large relative aperture.

In general, a good all-purpose lens having a small relative aperture will have a much better field than one with a large relative aperture, other factors being the same. That is, the aforementioned defects will be less pronounced in the image formed by the lens of smaller aperture. Moreover, the average amateur photographer *never* needs a lens with an aperture larger than $f/3.5$, and such a lens is more suited to his general needs than would be a lens of larger aperture. These two statements form the basis of my solution to the problem of acquiring a lens of long focal length which is at the same time good and inexpensive.

Fig. 1. Axial cross section of the RR lens, showing optical arrangement.

An older type of photographic objective known as the "Rapid Rectilinear," or RR, lens is shown in Fig. 1. It consists fundamentally of two identical cemented achromatic doublets mounted symmetrically about a central aperture stop. Owing to the symmetrical construction, distortion, coma and chromatic difference of magnifi-

cation are absent or greatly reduced. The two elements are individually corrected for spherical aberration and longitudinal chromatic aberration. Astigmatism is largely removed by proper placing of the stop. The RR lens thus produced includes an angle of view of 45° at aperture $f/8$ and has excellent definition in the center of the field. This type of lens was widely used in the best Kodaks and other cameras about the turn of the century.

If such a lens, say one of 5-in. focal length, is used with a Leica camera, the field of view covered is about 20°; thus only the best part of the field of the lens is involved, and the result is an excellent well-corrected objective of maximum relative aperture $f/8$. For many, perhaps most, purposes this lens would serve very well, for a lens used at aperture $f/8$ on a hand camera is fast enough for practically all purposes if modern emulsions are used, and is fast enough for Kodachrome if used out of doors in brilliant sunlight.

In case some may have more than academic interest in this matter, other remarks may be welcome. Figure 2 shows an RR lens of focal length 6 in. which came from a $3\frac{1}{4}\times5\frac{1}{2}$-in. folding Kodak dated 1902. This lens was made by the Bausch and Lomb Optical Company and was in excellent condition. I removed the entire shutter mechanism and rebuilt the lens, but this, of course, was not necessary. The original mounting could have been used, with considerable saving of labor, but the appearance would have been less pleasing. Photographic tests of this lens, mounted as shown, indicate that the field is good enough for general use. Among other ordinary tests, the most useful one was to photograph a sunlit brick wall partly covered with green vines. This was done with Kodachrome, from a distance of 50 ft at full aperture ($f/8$). When the picture was projected to a size of $3\frac{1}{2}\times5\frac{1}{2}$ ft on a screen, the color rendition was excellent and there was no evidence of aberrations. This would seem to be a severe test, certainly a utilitarian one.

The resolving power of this lens was determined by photographing a specially engraved chart.[1] In adjusting the lens relative to the chart,

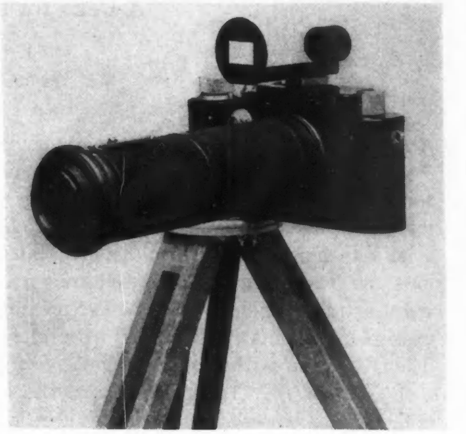

FIG. 2. Showing the RR lens mounted on a Leica camera.

it was set according to its focusing scale, and then placed at the corresponding distance from the chart. Thus in this test the lens was adjusted as it would be in everyday use. This investigation showed that the resolving power in all parts of the field ($1\times1\frac{1}{2}$ in.) was better than 20 lines/mm, and at this figure was equally good for both tangential and radial lines. It is perhaps unnecessary to point out that a more modern lens of the anastigmat variety commonly found today on the medium priced folding cameras might yield a somewhat higher resolving power.

Of course, a lens fitted in the afore-described manner will not function with the built-in range finder of the Leica camera (nor will the Leica telephoto lens of comparable focal length). Focusing is accomplished in the mounting shown by sliding the tube supporting the lens in or out of the main lens barrel. It is thus necessary to put a focusing scale on the lens barrel. To do this, it should be noted that the reference plane of the Leica is the flat collar on the front of the camera against which all lenses fit closely. When the lens is ready for marking the focusing scale, it will be mounted in its tube (which, in use, screws into the camera front). A piece of ground glass sheet is supported at the proper distance beyond the threaded end of the lens barrel, preferably attached rigidly by means of a short tube, and the lens is trained on an object at a known distance from the ground glass screen. When sharp focus

[1] Described by I. C. Gardner, "A test of lens resolution for the photographer," Bureau of Standards Cir. C428; this circular, with the engraved chart, can be obtained from the Superintendent of Documents, Washington, D. C., for 40 cts.

has been achieved, preferably by the parallax method, the setting of the lens slide may be scratched on the barrel, or otherwise marked. The procedure is the same for other distances chosen. If a range finder of known accuracy is handy, it is a simple matter to locate objects at specified distances. It is interesting to note in this connection that the lens position for an object at 100 ft and one at infinity (say $\frac{1}{2}$ mi) are noticeably different. Practically, however, this is of importance mainly in relation to the depth of field. For example, a certain lens, focal length $5\frac{1}{2}$ in., used at aperture $f/8$, specified circle of confusion, should have a depth of field 100 ft to infinity with lens setting infinity, and a depth of field 50 ft to infinity with lens setting 100 ft.

In order to save labor, a reasonable procedure would be to carefully mount the lens in a temporary but stable mounting and test it before spending much labor on it. If the focal length of the lens is too long, the opening in the camera front will limit the field. For the Leica camera, $5\frac{1}{2}$ in. is about the maximum focal length acceptable.[2]

[2] For a discussion of the technical and photographic aspects of the subject of photographic optics, together with extensive lists of references, see: A. C. Hardy and F. H. Perrin, *The principles of optics* (McGraw-Hill); K. Henney and B. Dudley, ed., *Handbook of photography* (McGraw-Hill).

An Experimental Method for Determining Coefficients of Sliding Friction

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MOST textbooks which deal with the subject point out the fact that coefficients of sliding friction may vary through wide ranges, even for the same types of surfaces. While no attempt will be made to discuss all of the reasons for this variation, some of it may be caused by the methods used in making the measurements.

One of two methods is generally used in student laboratories in determining coefficients of sliding friction. The first method consists of dragging a block of known weight with constant speed over a horizontal surface and observing the horizontal force required. The second method is to incline the surface and to find the angle at which the block slides with a constant speed.

Obviously there are several possible sources of error inherent in these methods. One of the most serious lies in the determination of when the speed of the block is constant. This is because of the fact that, in general, most surfaces are not uniform and the friction varies from place to place along the surface. Because of this variation, the difficulty is one of deciding when the *average* speed is constant, in order that an *average* value of the coefficient can be determined. In the present experiment (Fig. 1), this source of error is avoided by eliminating the need for moving the block with constant speed. Thus, it also avoids the possibility of getting anything other than an *average* value.

DESCRIPTION OF THE METHOD

The method proposed here combines some of the elements of an Atwood machine problem with those of the first method mentioned, that of dragging a block over a horizontal surface. The block of mass m_1 is connected to a driving weight of mass m_2 by means of a light cord which passes over a pulley whose friction and moment of inertia are as small as possible. The driving weight is allowed to move downward a distance h, so that the block and driving weight, just before the latter strikes the floor, acquire a speed v. The block will continue to slide along the horizontal surface for a distance d that depends upon the speed v and also upon the friction between the surfaces which are in contact. When the two masses m_1 and m_2 are equal, the distances h and d are the only quantities that need to be known. They may be measured with any suitable measuring rod.

THEORY

While the mass m_2 is falling, the forces involved are given by

$$m_2g=\mu m_1g+(m_1+m_2)a,$$

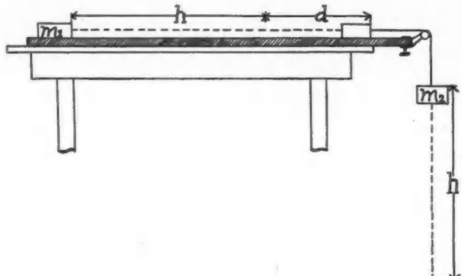

FIG. 1. Apparatus for determining coefficients of sliding friction.

where μ is the coefficient of sliding friction; therefore

$$a=\frac{m_2-\mu m_1}{m_1+m_2}g. \quad (1)$$

The energy equation is

$$m_2gh=\mu m_1g(h+d)+m_2ah. \quad (2)$$

Eliminating a between Eqs. (1) and (2), and solving for μ, we have

$$\mu=\frac{m_2h}{m_1h+(m_1+m_2)d}. \quad (3)$$

If $m_1=m_2$, the formula becomes

$$\mu=\frac{h}{h+2d}. \quad (4)$$

Equations (3) and (4) can be used as working formulas. It is seen that if the masses are equal, it is necessary to know only the distances h and d.

These formulas can also be derived by making use of the following expressions in succession: Eq. (1); $v^2=2ah=2Ad$, where A is the acceleration of the block after the driving weight has hit the floor; and $\mu m_1g=m_1A$.

TABLE I. Wood on wood.

h (cm)	d (cm)	μ, from Eq. (4)
58.5	27.9	0.51
58.5	27.0	.52
58.5	25.9	.53
58.5	26.4	.53
58.5	26.3	.53
58.5	27.3	.52
58.5	26.4	.53
58.5	26.1	.53
$f=130$ gm,	$w=247.4$ gm,	$\mu=f/w=0.52$

TABLE II. Wood on glass.

h (cm)	d (cm)	μ, from Eq. (4)
30	39.7	0.27
30	38.5	.28
30	36.8	.29
30	40.1	.27
30	39.8	.27
30	39.5	.28
$f=65$ gm,	$w=247.4$ gm,	$\mu=f/w=0.26$

EXPERIMENTAL DATA

Tables I and II will give an idea of the values that were obtained by the use of Eq. (4) as compared with those obtained by using the formula $\mu=f/w$, where f is the force needed to move the block along a horizontal surface at constant speed and w is the weight of the block.

Among the advantages of the method, in addition to those already mentioned, is the fact that, with a given pair of masses, measurements can be repeated rapidly on the same or different surfaces, a feature of value in those cases where it is desirable to have the measurements repeated by different persons.

The most fundamental effect of the industrial revolution on the common man was the proof that change was possible, and that it was not necessarily a calamity. . . . We owe to the experimental method this remarkable metamorphosis of the human spirit which in the brief span of two centuries turned a herd of abject slaves into a militant class that aspires to partnership with destiny.—TOBIAS DANTZIG, *Aspects of Science.*

Some Reasons why Physics Is Elected by so Few Freshmen Students; Suggested Remedial Measures

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"THE Board of Review of the Commission on Colleges and Universities in 1942, selected the equipment and offerings in the natural sciences as an area for investigation."[1] The study naturally brought up the old problem: Why is it that in practically all of our colleges and universities elementary physics is elected by so few freshmen students in comparison with the registration in freshman chemistry? After reading the report of the study, it occurred to the writer that while faculty and administrators had voiced opinions, student thought on the question was left untapped, and might hold the answer. It is the purpose of this paper to present a summary of such opinions along with suggested remedial measures.

The high school transcripts of 750 women students at present enrolled at Mundelein College revealed that 147 had credit in high school physics. Meetings were arranged with small groups of these students, and their cooperation was invited in filling out a questionary which had been prepared by the writer. Satisfaction in being consulted was expressed by the participants and, since no identification was required, the data may be considered unbiased and exact. Owing to the abundance of material collected through the questionary, only one phase will be reported on at present. Student opinion on items pertaining directly to the physics teacher such as his teaching load, student interest, subject interest and methods of presentation, was very appreciative as well as revealing. This material will be used in a further pursuit of the subject to discover which difficulties, as seen by students, are inherent in the subject and which have to do with generally used methods of presentation and teacher qualifications. Since the participants were not limited to giving a single reason for low registration in physics among college freshmen, but were invited to list as many reasons as they considered important and to number them in the order of their importance, the sum of the percentages in the following list exceeds 100 percent.

A survey of the results obtained from the questionary reveals the following points:

(i) 59 percent claimed that physics was too difficult;

(ii) 52 percent of the women students expressed the opinion that the profession of physics is definitely closed to them in normal times—that it does not offer women the opportunities that chemistry and biology do;

(iii) 41 percent said that physics required too much mathematics;

(iv) 28 percent found other sciences easier and more practical;

(v) 15 percent considered that most students do not know enough about the nature and purpose of a physics course to be interested in it;

(vi) 15 percent found that physics has too little to do with their major subject;

(vii) 8 percent contended that the deficiency in enrolment is attributable to inadequate vocational guidance for college students; other reasons closely related to this were expressed by 6½ percent as "not enough information is supplied about the type of positions available to a physics major after graduation," and by another 6½ percent as "the field of physics is not emphasized or encouraged in high school while biology and chemistry are;"

(viii) 8 percent believed that students are frightened by the alleged abstractness of the subject;

(ix) 5 percent pointed out that there is a "follow-the-leader" attitude among college freshmen.

All the reasons offered are of sufficient importance to keep enrolments in physics much below the capacity of the physics departments, but only a few are beyond remedy. The opposition arising from insufficient information about positions in physics available after graduation might be overcome if an attractive brochure were to be prepared by a group of physicists noted for their student interest. It might be circulated by the American Association of Physics Teachers and the American Institute of Physics, and should be made available to prospective college students during their senior year in high school or, at the latest, during the time between high school graduation and college registration. A

[1] A. J. Carlson, "The offerings and facilities in the natural sciences in liberal arts colleges," North Central Assoc. Quart. 18, 154–64 (1943).

single teacher attempting to undertake the project would undoubtedly be charged with high pressure advertising for his department.

Another suggestion concerns the production of motion pictures in which persons of note and representatives of various divisions of physics might be interviewed while at their work. Scenes taken at centers of physics research and in industries located in various parts of the United States could be included in the same film or made into another short. These films should be made available to physics teachers and the general public just as commercial films are.

To remove the objection that physics is too difficult and requires too much mathematics, an introductory course designed exclusively for non-science majors should find its way into the curriculums of more colleges. Its purpose is to stress the applications of physics to the life of the liberal arts student. This course embodies experiments and problems chosen on the basis of general application rather than preparation for specialization in physics.

Several universities have introduced lecture-demonstration courses to this end, but such a procedure deprives the student of one of the most important disciplines for which physics is outstanding—that of manipulation and exact measurement. Lecture, demonstration, laboratory work and problem solving—all are needed for satisfactory training in physics whether intended for specialization or for general knowledge. It is not a matter of omission but of adaptation and selection of material.

Other remedial measures such as active local physics clubs, student participation in projects, press notices of departmental activities, and so forth, are but a few that many teachers have already used. However, it seems to the writer that since the situation which faces us is national, a remedial plan also national in scope should be adopted.

Industrial Experience for College Teachers

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SEVERAL articles have appeared in the AMERICAN JOURNAL OF PHYSICS during the past several years discussing the training of physicists for industrial work and research and suggesting curriculums for such training. College teachers and industrial physicists alike appear to recognize the need for certain changes in present methods of training physicists for industry, and improved curriculums will undoubtedly be devised.

College physics teachers are often accused of being too theoretical and not bringing a sufficiently practical viewpoint to the subject which they teach; they have been prone to let the engineering professors teach the practical applications of physics. This has some advantages, but it may lead to a lack of understanding of fundamental theory by engineering students. This separation of function may be due, in part, to a lack of mutual understanding of viewpoints between physics teachers and teachers of engineering subjects. Professor Knudsen has said,[1] "Physics, especially in the universities, has stayed too aloof from engineering, and engineering has strayed too far from physics." This viewpoint has been impressed upon the writer from his association with young engineers in an aircraft engine plant. Some of these men have expressed disappointment that their physics professors had not had some industrial experience so that they might have pointed out more practical applications of the subject.

Doctor Littleton[2] has pointed out that one-half of the physicists in the United States are working in industry, and that this proportion will probably increase. Yet in spite of this fact,

* Present address: Raytheon Manufacturing Company, Newton, Massachusetts.

[1] Am. J. Phys. 11, 74 (1943).
[2] Am. J. Phys. 11, 316 (1943).

physics curriculums are designed principally for the training of college and university teachers, and not of industrial physicists.

In view of the fact that there are at present only a relatively few graduate students in the field of physics who are preparing to be teachers, and that, for one reason or another, many regular teachers are not in a position to take courses relating to the many industrial applications of physics, the writer would like to suggest that plans be devised whereby teachers may obtain practical industrial experience.

The Pratt & Whitney Aircraft, Division of United Aircraft Corporation, at East Hartford, Connecticut, with whom the writer has been associated for three summers, began several years ago to employ college professors and instructors in their Engineering Department during the summer months. This program was intended to provide these teachers with industrial experience which would aid them in training students, some of whom might later be associated with the industry, and at the same time to secure for the company the benefits of a fresh viewpoint and academic experience in delving into the more complicated and theoretical engineering problems. The number of professors employed was increased annually until last year, when the demands of the armed services and the effects of accelerated college programs reduced the number of men available. In 1942, the peak year, 25 teachers, representing 19 colleges and 12 states, were employed.

It is suggested that many other industrial organizations, both large and small, might adopt a plan similar to that of Pratt & Whitney Aircraft which should benefit the industries and also those teachers who are interested in obtaining practical industrial experience. Such a plan would give physics teachers an opportunity:

(1) To learn, at first hand, the types of problems facing industry, the practical applications of physics to their solution, and the limitations, or approximations necessary, in applying physical principles to production methods.

(2) To discover weaknesses in present methods of training industrial physicists and engineers, and thus to improve their own curriculums and methods of teaching.

(3) To work with young physicists and engineers, learn their viewpoints and share experiences with them.

(4) To meet and to share experiences with teachers from other institutions.

(5) To learn the uses of physical apparatus in making industrial measurements and tests.

(6) To observe and perform research, which in turn might lead to an interest in some specific research problem that might be continued at the teacher's own institution.

(7) To travel and live in a different section of the country for a few months.

(8) To provide an income for the summer in a congenial and interesting type of work.

(9) To obtain a permanent position with an industrial organization, if a teacher eventually decided that he was better qualified for, and preferred, that type of work.

Some advantages to an industrial organization would be:

(1) A better understanding of, and a more sympathetic attitude towards, the technical and economic problems of industry in general, and of one industry in particular, on the part of the teachers.

(2) Association of their young engineers and physicists with men of higher academic training and wider experience; this should help to remind the younger men of the importance of continual study and reading in their particular field.

(3) Provide in the future better trained engineers and industrial physicists, as their teachers would be more familiar with the problems of industry.

(4) Provide additional trained men to work on engineering and research problems; short problems could be devised, or longer ones initiated, which might be continued at the teacher's own institution.

(5) Provide, incidentally, good advertising for the products of a particular industry.

The benefits gained, although not always capable of expression in dollars and cents, should outweigh the relatively small expenditure for salaries.

If such a program is to be carried out successfully, it must be efficiently organized. The American Institute of Physics or the American Association of Physics Teachers might act as a clearing house for interested teachers and employers. Industrial physicists and engineers could prepare lists of possible problems in their respective organizations, and these lists could be made available to the prospective teachers so that they might negotiate with an industry having a problem in which they were interested and qualified to work. Salary schedules should also be made available to the teachers. It might be advantageous to have the salaries as uniform as possible.

Should such a plan for industry prove practical, it might also be adopted by hospitals and medical laboratories, where teachers of pre-

medical students could obtain valuable experience. Summer work in other closely related fields might also materialize.[3]

[3] The writer would like to suggest that physics teachers interested in such a plan send additional suggestions and criticisms to the Editor of the *American Journal of Physics* or to the Director of the American Institute of Physics.

Appreciation is expressed to Pratt & Whitney Aircraft and to the summer professors employed by the company for making available information concerning the company's summer program, as the ideas expressed have furnished a basis for some of the suggestions made in this paper.

Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

21. Three Prophetic Aeronautical Prints

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ATTENTION has been called recently to a number of interesting predictions concerning the invention, use and misuse of aircraft.[1] Such predictions are really not particularly surprising if written in the nineteenth century because, as a result of the lighter-than-air developments which took place during the last two decades of the eighteenth century, almost all of the uses—and misuses—to which airplanes have been put during the twentieth century were seriously suggested and even tried with balloons, and many of these suggestions and trials not only were recorded but were given widespread publicity by the numerous prints and caricatures which appeared during that period.

The prints reproduced in this article are three examples from many that might have been chosen to illustrate the military uses of aircraft which were suggested about 1800. Except for the fact that balloons are depicted instead of airplanes, they might almost represent battle scenes of the present war.

The first suggestion of the possible military use of aircraft was made in 1670 by the Jesuit, Francesco Lana (1631–1687), in connection with his proposal of the first lighter-than-air

[1] See, for example, M. F. Ashley Montagu's note in Science **98**, 431 (1943).

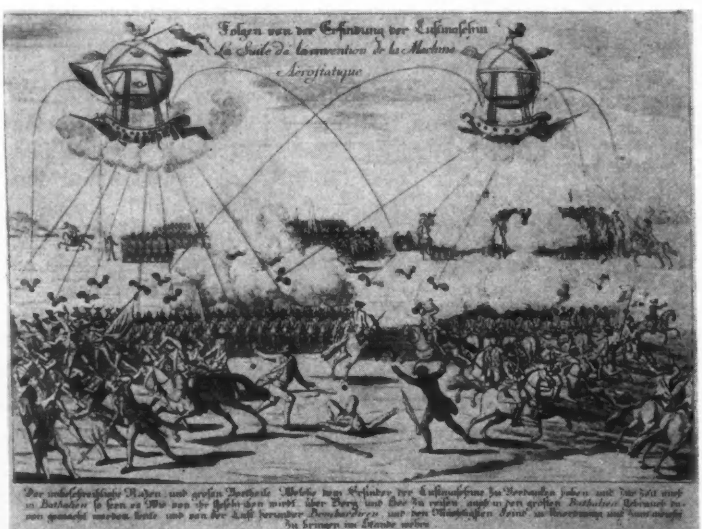

Plate 1. A German caricature showing two military balloons strafing troop concentrations. (*Circa* 1804.)

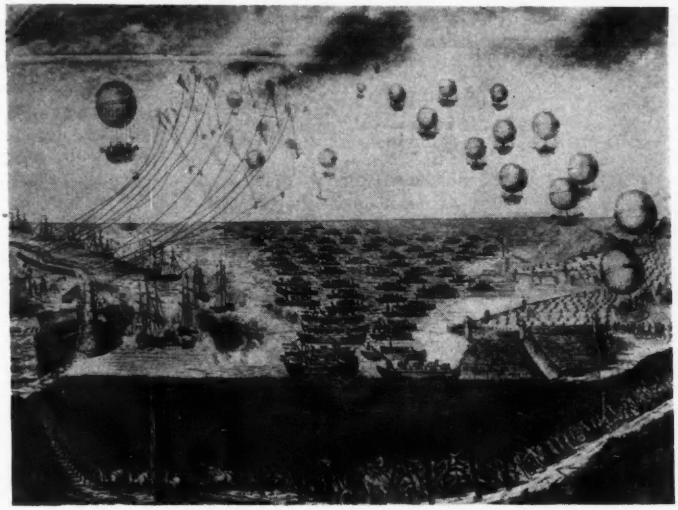

PLATE 2. A Napoleonic project for attacking England. (*Circa* 1803.)

machine based upon a sound scientific principle.[2] LANA did not attempt to construct the proposed aerial ship for the following reason:

> God would not suffer such an invention to take effect, by reason of the disturbance it would cause to the civil government of men. For who sees not that no City can be secure against attack, since our Ship may at any time be placed directly over it, and descending down may discharge Souldiers; that the same it would happen to private Houses, and Ships on the Sea: for our Ship descending out of the air to the Sails of Sea-Ships . . . it may over-set them, kill their men, burn their Ships by artificial Fire-works and Fire-balls. And this they may do not only to ships but to great Buildings, Castles, Cities, with such security that they which cast these things down from a height out of Gun-shot, cannot on the other side be offended by those from below.[3]

This is a truly remarkable prediction and one that has been completely realized during the present war. That the possibilities envisioned by LANA and achieved by modern war planes were also considered very seriously nearly 150 years ago is clear from the three prints here reproduced.

The bombing and strafing of troop concentrations by airplanes has become standard practice during the present war. Plate 1 is a reproduction of a German caricature of about 1804 showing two armies, one of which is put to flight by the artillery of two aircraft.

During the Napoleonic Wars several schemes for attacking England from the air were proposed. Plate 2 shows the French forces attacking in balloons, in barges and through a channel tunnel. Actually from May 1803 until August 1805, NAPOLEON had an army of approximately 130,000 men encamped at Boulogne while more than 2000 flat-bottomed boats and barges, designed to land men and equipment on the beaches at Dover, were constructed and assembled. The project was abandoned only shortly before the Battle of Trafalgar.

Plate 3 is an early (*circa* 1803) suggestion of a bombing attack from the air on the British Fleet.

Within the last 140 years three major attempts to conquer the world have been made, the first by France, the second and third by Germany. Each time the attempt has been thwarted largely by the British. It is remarkable what a controlling part in world history has been played by one staunch little island.

PLATE 3. A Napoleonic project for destroying the British fleet. (*Circa* 1803.)

[2] F. Lana, *Prodromo overo Saggio di alcune Inventioni Nuove premesso all'Arte Maestra* (Brescia, 1670).

[3] Translation by Robert Hooke, *Philosophical collections* (London, 1679), No. 1, p. 18.

NOTES AND DISCUSSION

Resolving Power

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THERE is a common misconception that the term resolving power has meaning when applied to the observation of a single particle. The fact is that complete resolution is the distinct separation of two images and necessarily implies the existence of at least two corresponding particles.

A reference to the resolving power of the unaided eye will emphasize the distinction, for the same principles apply in this case as in observations with instruments. If two stars subtend an angle appreciably less than one minute of arc—the resolving power of the eye—they cannot be seen as two stars. The eye does, however, permit us to see single stars, all of which subtend much smaller angles, as for example, Betelgeuse, which, according to Michelson's measurements, subtends an angle of only 0.047 *second* of arc. Moreover, this first magnitude star is "clearly and sharply defined" and is certainly "vivid," using one recent author's description of a resolved image. Nevertheless, this tells us nothing about the resolving power of the unaided eye.

The requirement for seeing single particles smaller than the limit of resolution, with or without a microscope or telescope, is merely that of sufficient contrast with respect to the background. As is well known, this principle finds useful application in ultra-microscopy, where also the smallest particle size discernible has nothing to do with the resolving power.

It is unfortunate that so important a concept as resolving power is frequently misunderstood by microscopists and others. Perhaps a greater emphasis by physics teachers and textbook writers on the principles and derivation of the formula for resolving power would show the exact meaning of the result and prevent the perpetuation of this misconception.

Dalton's Law of Vapors

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DALTON'S law may be expressed by saying, "The maximum pressure exerted by the vapor of a particular liquid in a closed space at a definite temperature depends only on that temperature, and is independent of the pressure of other vapors or gases having no chemical action (including solution) on it."

A common experiment on the lecture table is to saturate the empty space above the mercury in a barometer tube with ether vapor, measure the depression of the mercury column and so obtain the value of the saturation vapor pressure of ether at room temperature. In a companion experiment, air occupying a known volume at atmospheric pressure is saturated with ether vapor. This raises the pressure and also increases the volume. The volume occupied by the air and ether vapor is then restored to its original value, when it is found that the pressure of the contents now exceeds atmospheric pressure by an amount very nearly equal to the saturation vapor pressure of ether when ether vapor alone is present.

In another form of apparatus described in some textbooks,[1] a big bottle is fitted with a large stopcock through which passes the lower end of a siphon barometer, a tube from a thistle funnel and a tube for evacuation of some of the air. First the pressure of the air in the bottle is reduced to about 30 cm of mercury, then ether is introduced and one hopes that the pressure will rise to the required value. I find, however, that the process of reaching saturation is too slow to make this an effective experiment for the lecture table. An advantage of this form of apparatus is that a change in volume is not involved.

A note in Ganot's *Physics* says that Regnault tried experiments of the kinds just described and remarked that the total pressure of the mixture was rarely up to the value expected, but he thought it near enough so to make Dalton's law acceptable.[2]

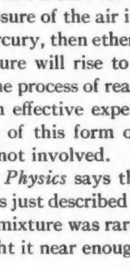

FIG. 1. Dalton's law apparatus.

These experiments are well worth doing, and ether is usually selected as the liquid because its vapor pressure at room temperatures is of the order of 40 cm of mercury, so that the class can see what is happening. The chief difficulty is that as the experiment is usually carried out, the ether is introduced into the tube through stopcocks. My experience is that when the volume of gas in the tube is brought back

to its original value, the ether escapes through the stopcocks, because the usual tap grease is soluble in ether. The escaping ether takes air with it, and it is impossible to show that the vapor pressure of the ether in the presence of the air is about 40 cm of mercury at ordinary room temperature.

The apparatus shown in Fig. 1 is very effective in its method of introducing the ether and certainly cannot leak. The narrow tube T effects the improvement. The tube DC is about 1 ft long, and BD is about 3 ft long. The reservoir R and the tubes contain mercury. The tube CDB and the reservoir R, both suitably supported, are manipulated so that air is got through the mercury into CD and at atmospheric pressure has a volume of, say, CA. It is quite easy to get A and A' at the same level. The point A is marked with a piece of gummed paper.

Ethyl ether is then poured in at B and, by lowering R and suitably inclining CDB, a little ether is run from the A' side into the A side. This process is well under control. Immediately R is raised to prevent a backflow of the mercury as the ether vaporizes. The ether in AC now evaporates and in time saturates the air; and if R be now raised to bring the mercury in CD back to the gummed paper, the mercury in BD rises and the vertical distance between A and E will be of the order of 40 cm.

The surprising thing is the length of time required by the ether to saturate the air. When one carries out the first experiment (ether alone above the mercury in a barometer tube) the process is complete in a few seconds. But with air present the ether pressure builds up surprisingly slowly, reaching only 20 cm of mercury, say, in 10 min, nearly 40 cm in 20 min and its final height (40.0 cm at 20.0°C) in about a day. This certainly reminds one of the delay caused by molecular collisions.

An important point is that the setting of the surface of the liquid ether in AC to the original air volume must be as accurate as possible. The pressure is measured of course from E to the mercury surface, which is now just below A; the correction for the liquid ether at A is not worth making. Temperature must be kept constant all the time, or else the tube CD must be jacketed.

The design of this apparatus is almost wholly owing to Mr. Chappell, the department glass blower.

[1] For example, T. Preston, *The theory of heat* (Macmillan, Ed. 4, 1929), Chap. 5, Sec. 5.
[2] See also reference 1.

A Method of Ruling Equidistant Parallel Lines

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THE following method for ruling parallel lines, which has been used for some years by the writer, may prove of interest as a supplement to the article on diffraction gratings appearing in this journal.[1] Essentially the method consists of moving a pen or marking device across the surface to be ruled by means of the lead screw of a lathe. The surface is wrapped around an arbor, and a pen attached to the carriage draws a continuous line on the surface. If a few precautions are taken, excellent rulings may be made in this manner.

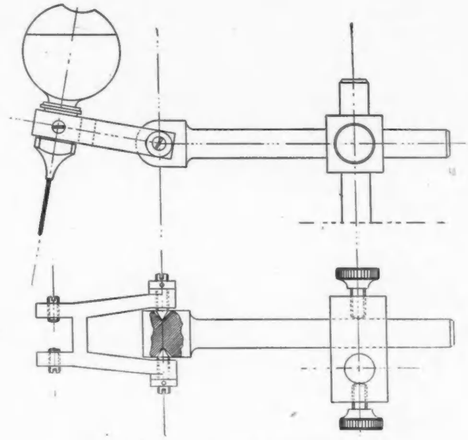

FIG. 1. Method of ruling parallel lines.

A satisfactory pen is made from a Yale B-D, 0.020-in., hypodermic needle. The needle is ground approximately square to remove the bevel, and a glass reservoir is attached to the syringe end. This may be a glass bulb or tube with a capacity of about 5 cm³. It is waxed in place with Picein. For ruling very fine lines the hypodermic needle may be replaced by a glass tube drawn to a capillary. This assembly is held in a yoke with set screws as shown in Fig. 1 and adjusted so that the pen moves up and down very freely, but has no lateral motion.

It will be found convenient to use a large arbor to facilitate the mounting of the surface to be ruled. A metal tube 6 in. in diameter and about 24 in. long is an arbor of reasonable size. The surface to be ruled is attached to the arbor with Scotch tape. The seam is not objectionable because the pen, if adjusted properly, will ride over it and not more than half an inch of the ruling will be wasted. Since finger prints and oil spots show up in relief in the final ruling, it is necessary to have both the arbor and the surface absolutely free of oil.

The yoke carrying the pen is attached to the carriage tool post and adjusted so that the pen rides approximately at the top of the arbor at an angle of about 10° or 15°. The pen is swung out of position, and the surface to be ruled is attached. If black rulings are to be made, the pen is filled with Higgins' Eternal Black Writing Ink. If the ink does not drip freely from the pen, it may be started by pushing a fine wire through it.

The lathe is geared to give a surface speed of about 20 ft/min to the arbor and is started in the reverse direction. The pen is allowed to rest on the surface and, as soon as the line starts, the lead screw is thrown in and the pen is allowed to finish the run.

The first ruling will show many irregularities in width and density of line. These will disappear as the pen be-

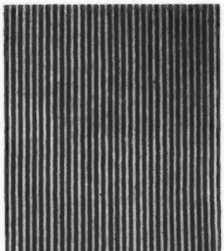

FIG. 2. Small portion of an 18×24-in. ruling (×1).

comes polished and takes the form of the arbor. It has been found that the most satisfactory surface for these rulings is a single-weight, semigloss drawing Bristol. Blue tracing cloth, good grades of tracing paper of various weights and some writing paper will also give good rulings. It is advisable to test a sample of the surface about 2 in. long by attaching it to the arbor and ruling it. Inspection with a 2-in. lens should show a line with very sharp edges, and the first and last lines should have the same density. The pen should be washed after each ruling with warm water to which a little soap or ammonia has been added.

One ruling, 18×24 in. having 820 lines, with a paper centimeter scale attached, was photographed lantern slide size. Several reductions were made to about 800 lines per inch for lecture demonstrations. Crossed gratings were made either by making crossed rulings or by crossing two negatives. Plotting paper has been made by ruling a surface, turning it through 90° and ruling another set with the same or different spacings.

If the lathe has any periodic or progressive errors, inspection of the ruling will show them very distinctly and the writer has used this method on occasion for such tests. Rulings made on tracing paper and superimposed will show very interesting interference effects.

[1] Schultz, "Diffraction gratings at low cost," Am. J. Phys. 12, 105 (1944).

Propagation of Energy—A Lecture Demonstration

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A VERY common but, to a beginning student, a rather abstruse phenomenon is the propagation of energy from the point at which the work is done to some other point. This transfer of energy may be accomplished by many different means. In a locomotive the steam does work on the piston head, but the energy is propagated by a connecting rod to the driver where it is used to do work and move the locomotive. In an automobile the situation is similar; work is done on the piston by the products of combustion, but this energy must be propagated by the connecting rod to the crankshaft and thence via the gears, driving rod and axle to the wheels.

In the apparatus diagrammed in Fig. 1, work is done on the piston while the energy appears in the other U-tube as a change in the difference of the water levels. This energy has evidently been propagated by the gas in the connecting tube A. The air column in the tube, as a conveyor of energy, acts in a manner analogous to the connecting rod and other connections of the preceding examples.

So far the experiment has been very naive, and nothing has been shown that the student did not know before. It is at this point that the instructor emphasizes the profundity of the principle in question by inserting in place of the heretofore short connecting tube a piece of rubber tubing 100 ft or more in length. Now when the piston is moved, the energy imparted to it is again transmitted to the water in the second U-tube. But this time the beginner is usually astonished to find that the water in the second U-tube does not move in phase with the piston. As a matter of fact, with this apparatus, if the piston is moved up and down at about 4 vib/sec, the water will oscillate out of phase with the piston. This effect has been used by the author in lectures on sound to illustrate that (i) energy can be transmitted from one point to another by an elastic medium, and (ii) energy is always transmitted with a measurable, finite velocity.

The apparatus can be readily used to measure the velocity of transmission of the energy. This energy is propagated through the air column by means of a longitudinal wave of very low frequency and very long wave-

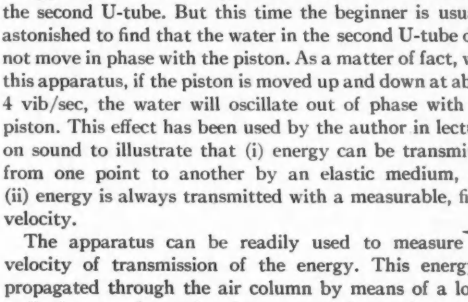

FIG. 1. Apparatus for demonstrating propagation of energy.

length. When the water and the piston oscillate exactly in phase opposition, a standing wave has been set up as if the tube were an open air column of total length $\frac{1}{2}\lambda$; this means that the wavelength is about 200 ft. Since the frequency is about 4 vib/sec, the velocity c is about 800 ft/sec. It is

noteworthy that the velocity obeys the isothermal law first propounded by Newton, $c=(p/\rho)^{\frac{1}{2}}=(RT)^{\frac{1}{2}}=980$ ft/sec, probably because in such a long tube of small diameter the ratio of the tube area to tube volume is large so that a rapid transfer of heat into or out of the transmitting medium is possible.[1,2] These latter points are best reserved for a later discussion. It is not wise to distract attention from the first two profound principles.

A demonstration similar to this one but of particular interest to premedical students has been described by Little.[3]

* The assertions contained herein are the private ones of the writer and are not to be construed as official or reflecting the views of the Navy Department or the Naval Service at large.

1 H. Helmholtz, Crelles J. **57**, 1 (1859).

2 A. B. Wood, *A textbook of sound* (Macmillan, 1937), p. 238.

3 N. C. Little, Am. J. Phys. **6**, 30 (1938).

Models as Aids in Calculation

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INVENTORS of ingenious devices and demonstration models often publish in these pages. I should like to remind them of some larger problems deserving of their attention and inventiveness. These problems occur in those regions where physical theory is mathematically most complex in spite of the simplifications introduced by bold approximating, but where (as in astrophysics, in atomic and molecular domains, and in several borderline fields) ordinary scale models cannot be brought in to console the interpreter for the weakness of theoretical guidance. In such fields there is need for the design of new devices which will perform the normal functions of scale models, that is, which will by-pass inadequate theory and which will simplify the labor of solving difficult equations.

There are two types of such mechanical aids to calculation that manage to avoid the slavish imitation of the scale model as well as the abstractness of the calculating machine:

I. *Substitution models.* These may not only change scale but also materials used and properties observed; many such models are based on the formal similarity of laws in different fields. Examples:

1. Stresses in beams: stress-double refraction in a plastic.
2. Particle motions in a conservative field: rolling balls on a stretched rubber sheet.
3. Electron densities in a crystal: photographic images from manipulation of diffraction patterns.
4. Transportation costs (economics): resistances in slide-wires on a map.
5. Nerve action (neurophysiology): action of nitric acid on iron wire.
6. Electric current: water current.
7. Planetary motion: steel ball around a pole.
8. Nucleus: liquid drop.
9. Gas: elastic spheres.
10. Mechanical vibration: circuit oscillations.

Examples 8, 9 and 10 are mainly imaginary models for mental stimulation.

II. *Calculation models.* These simplify or illustrate computations in "pure" mathematics with the help of physical laws. Examples:

1. Euclidean geometry: compass-ruler construction.
2. Mathieu functions: electric potentials across an elliptical wave guide.
3. Product sums (correlation statistics): moments of inertia of mass points in a mesh.

Within their limits, such devices are evidently capable of quantitative predictions, and are worth inventing for this purpose when mathematics becomes so intricate, tedious or unfruitful that the distortions and approximations of the mechanical representation are preferable.

To illustrate how many things are waiting to be done in this line, let us take quantum theory as an example. Several mechanical analogies come quickly to mind which might be worth attempting to develop into quantitative models:

I. 1. "Cut-off" calculations: standing waves, acoustic or electric (dielectrics in cavity resonators).
   2. Two-dimensional wave equation: surface waves in a liquid with varying refractive index obtained by varying the depth.
   3. Choice of correct wave-function combinations: coupled pendulums.

II. 1. Factoring of secular equation: some similar macroscopic eigenvalue problem?
   2. Discovery of normal coordinates: mechanical analog?

Such models might be devised which would oversimplify less and distort less than some theoretical approaches to the same problems.

In fact, some models of these types could, like scale models, actually avoid certain coarse approximations necessary for theoretical treatment. To take a hypothetical case, it is patent that the many-body problem is repeatedly solved without approximation in nature—secular equations factored, degeneracies removed, and all!—and if we could set up such a case (for nature to solve) whose theory we think we understand completely, it might serve as a very accurate model for some other case whose theory is still in doubt and still being tested. Such a many-variable model does not seem so impossible to construct when one considers, for instance, the manifold dynamic relations which can be obtained among the component parts of an electric circuit, and the success of such circuits to date in solving differential equations and many-variable problems such as gun-directing.

There are opportunities in almost every field for inventive spirits to devise these more subtle models. Such "experimental" approaches to theory have been produced in the past by the historical accidents of invention, but they might well be made the objects of deliberate and systematic search. If there could be thus built up a body of such "computational" schemes with known advantages and limitations, it might enable physicists and border-physicists to avoid some of the tedious and approximate mathematics which sometimes delays the application of physical analysis to more complex realms.

A Picturable Derivation of the Coriolis Acceleration

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AN air of mild mystery seems to hover about the Coriolis acceleration (also called "composite" or "additional"), and a demand is often heard for a derivation whose every step has a picturable physical meaning. In other words, the derivation desired would not be based on coordinates and coordinate transformations. The following treatment, presumably not very widely known, permits the student to visualize the mechanism generating the Coriolis acceleration.

1. Let us first recall that the time-rate of change of a vector of magnitude X and angle ϕ consists of two parts: one caused by change in magnitude, and one caused by change in direction.

(*a*) The time-rate owing to change in magnitude is $\dot{X}$ in the direction of X ("longitudinal change").

(*b*) The time-rate owing to change in direction is $X\dot{\phi}$ normal to the vector in the sense of increasing ϕ ("transversal change").

2. Consider now a carousel rotating at a speed ω, and a particle whose velocity *relative to this carousel* has a radial component $p[=\dot{r}]$ and a transversal component $q[=r\dot{\theta}]$. All transversal components are measured in the direction of increasing θ, which is also the direction of increasing ϕ.

The *absolute* velocity of the particle will have the radial component p, but a transversal component $q+r\omega$. The absolute angular velocity of the particle is $\dot{\theta}+\omega$.

Any component of acceleration that is dependent on r, $\dot{r}$, θ, $\dot{\theta}$ only is a component of the relative acceleration. Any component that is dependent on r, ω, and $\dot{\omega}$ only is a component of the entrained acceleration, that is, of the acceleration of that particle of the carousel with which our first particle momentarily coincides. Components depending on variables belonging to both these sets are components of the Coriolis acceleration. It is in these that we are interested.

3. The absolute acceleration of the particle is owing to four causes:

(*a*) The change in magnitude of the radial velocity p. Its rate is $\dot{p}$ in the radial direction. This is a relative acceleration.

(*b*) The change in direction of the radial velocity p. Its rate is $p(\dot{\theta}+\omega)$ normal to p. Here $p\dot{\theta}$ is a relative acceleration, but *$p\omega$ is an additional acceleration*. It arises from a change in direction caused by the rotation of the carousel.

(*c*) The change in magnitude of the absolute transversal velocity $q+r\omega$. Its rate is $\dot{q}+r\dot{\omega}+\dot{r}\omega$, transversal. Here $\dot{q}$ is a relative acceleration, $r\dot{\omega}$ an entrained acceleration, but *$\dot{r}\omega[=p\omega]$ is a Coriolis acceleration*. It arises from a change in magnitude of the entrained velocity $r\omega$ caused by the relative radial motion.

(*d*) The change in direction of the absolute transversal velocity $q+r\omega$. Its rate is $X\dot{\phi}$ radially towards the center, or $-(q+r\omega)(\dot{\theta}+\omega)$ if measured positively away from the center, as is usually done. Expanding the latter expression, we get: $-q\dot{\theta}$, the relative centripetal acceleration; $-r\omega^2$, the entrained centripetal acceleration; *$-q\omega$ and $-r\omega\dot{\theta}$, which are Coriolis accelerations.*

The first one, $-q\omega$, is caused by the absolute change in the relative velocity q, caused by the rotation of the carousel; the second one, $-r\omega\dot{\theta}$, is caused by the change in the entrained velocity $r\omega$, caused by the relative angular speed $\dot{\theta}$. Since $q=r\dot{\theta}$, the two terms add to $-2q\omega$.

The Coriolis accelerations are listed in Table I.

TABLE I.

Components of absolute velocity	Components of the Coriolis acceleration caused by direction change brought about by entrained rotation	Components of the Coriolis acceleration caused by direction change brought about by relative rotation	Components of the Coriolis acceleration caused by magnitude change brought about by radial motion
p radial	$p\omega$ transversal		
q transversal	$-q\omega$ radial		
$r\omega$ transversal		$-r\omega\dot{\theta}=-q\omega$ radial	$\dot{r}\omega=p\omega$ transversal

Collecting all the Coriolis accelerations, we obtain: $2p\omega$ transversal and $-2q\omega$ radial. Each of these is obtained by rotating p, respectively q through a right angle in the direction of rotation, and multiplying it by 2ω. Their resultant is, therefore, obtained by multiplying the resultant of p and q by 2ω, and turning it 90° in the direction of rotation. This is the Coriolis theorem.

4. This discussion may be variously simplified for class use. Ordinarily ω is assumed constant. If generality is not the objective, only the cases of radial motion ($\dot{\theta}=0$) and transversal motion ($\dot{r}=0$) may be considered. With the resulting simplification, the generating causes of the composite acceleration stand out very clearly.

The familiar formulas for the polar components of acceleration,

$$a_r=\ddot{r}-r\dot{\phi}^2=\dot{p}-r\omega^2,\quad a_\theta=r\ddot{\phi}+2\dot{r}\dot{\phi}=r\dot{\omega}+2p\omega,$$

can be used to illustrate the Coriolis acceleration. They are a special case of the preceding discussion, with $\omega=\dot{\phi}$ and $\dot{\theta}=0$. This makes the radius our carousel, and a radially moving particle our particle. The term $2\dot{r}\dot{\phi}$ is thus shown up as the Coriolis acceleration, the radius being the moving system.

Note on the Paper "Resistances in Parallel"

H. L. DORWART
Washington and Jefferson College, Washington, Pennsylvania

BELL and Zabetakis[1] have shown how solutions of the equation

$$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2} \tag{1}$$

can be obtained very simply by setting

$$R_1/R_2=R_P; \tag{2}$$

then, for any value of R_P, R_1 and R_2 are calculated from

$$R_1=R_P(1+R_P),$$
$$R_2=1+R_P. \tag{3}$$

Probably this method gives ample solutions for the purpose of making up problems for a first course in physics, but it may be of interest to consider briefly the general solution of Eq. (1) in *integers*. In the theory of numbers, such a problem is referred to as the solution of a Diophantine equation.

The monumental *History of the Theory of Numbers*, by L. E. Dickson,[2] contains references to 21 papers discussing either Eq. (1) or its generalizations. The solution given in the next paragraph was known as early as 1895, but E. Sós in 1905 appears to have been the first to prove that it is the general solution.

One can quickly show by direct substitution that

$$R_P = kbc, \quad R_1 = kb(b+c), \quad R_2 = kc(b+c) \tag{4}$$

satisfy Eq. (1). If b and c are taken to be any two relatively prime integers, then Eqs. (4) are the general solution of Eq. (1).

As stated by Dickson,[3] if a solution with $k=1$ is called irreducible, and if

$$R_P = p_1^{a_1} p_2^{a_2} \cdots p_n^{a_n},$$

where $p_1, p_2, \cdots, p_n$ are distinct primes, and $a_1, a_2, \cdots, a_n$ are zero or positive integers, then there are 2^{n-1} essentially distinct irreducible solutions corresponding to a given R_P, with R_1, R_2 counted the same as R_2, R_1.

For $R_P = 1, 2, 3, 4, 5$ the only essentially distinct irreducible solutions are those given by Bell and Zabetakis in their Table I. However, for $R_P = 6 = 2\cdot 3$, $2^{n-1} = 2$, and there should be a second pair in addition to $R_1 = 42$, $R_2 = 7$, the only pair given by Eqs. (3). Taking $k=1$, $b=2$ and $c=3$ in Eqs. (4), this second pair is found to be $R_1 = 10$ and $R_2 = 15$. (The first pair is given by Eqs. (4) with $k=1$, $b=6$, $c=1$.)

If we do not insist on irreducible solutions, then the number of essentially distinct solutions corresponding to any R_P, as given by Dickson,[3] is

$$N = \tfrac{1}{2}\left[\prod_{k=1}^{n} (1+2a_k) + 1\right]. \tag{5}$$

Thus for $R_P = 6 = 2\cdot 3$, where $n=2$ and $a_1 = a_2 = 1$,

$$N = \tfrac{1}{2}[(1+2)(1+2)+1] = 5.$$

The solutions are

R_1	42	15	24	18	12
R_2	7	10	8	9	12

only the first two pairs being irreducible.

For $R_P = 30 = 2\cdot 3\cdot 5$, in addition to $R_1 = 930$, $R_2 = 31$, given by Eqs. (3), there are three more essentially distinct irreducible solutions, as follows:

b	c	$b+c$	R_1	R_2
6	5	11	66	55
10	3	13	130	39
15	2	17	255	34.

Here, the number of essentially distinct solutions is

$$N = \tfrac{1}{2}[(1+2)(1+2)(1+2)+1] = 14.$$

[1] R. M. Bell and M. G. Zabetakis, Am. J. Phys. 12, 231 (1944).
[2] Published by the Carnegie Institution of Washington, 1920.
[3] Reference 2, p. 690.

Vocational-Technical Training and Its Importance to Physics

Senate bill S, 1946, introduced in May 1944 by Senator George and now before the Committee on Education and Labor, authorizes the appropriation annually of $97,500,000 to be distributed among the states for vocational-technical training. The bill is an extension of the provisions of the Smith-Hughes Act of 1917 under which vocational training financed by Federal funds was begun. However, it is to a certain extent of an emergency nature, since it is intended for veterans returning from military service and workers demobilized from war industries, as well as other youth and adults. After 1946 the states must match 25 percent of the money they receive to be eligible under the provisions of the bill.

The bill is vague in many respects. It creates "area schools organized for the purpose of conducting vocational and/or vocational-technical training of less than college grade" without defining the meaning of such terms as "area school" and "college grade." Some clarification of the meaning of these and other terms is needed. The administration of the training provided by this bill is to be in the hands of the state boards for vocational education, which were set up under the Smith-Hughes Act and which include only representatives of labor and of management.

Experience in Army and Navy training classes, in the ESMWT program and in industrial training schools shows clearly that the most more valuable results are achieved if training is for a field of related occupations than where it is only for a particular job. It has also become clear that a foundation must first be laid on which to build such training. It is essential that the bill now before Congress should specifically state that the proposed schools must give not only vocational-technical training but also the mathematics and the sciences basic thereto. Chief among these sciences are, of course, the physical sciences, and it is of the greatest importance that physics be recognized as basic to all such proposed training for engineering application. This has been recognized fully in the ESMWT program, through the appointment of a physicist on the administrative staff of the U. S. Office of Education. The schools and classes provided for by the bill would offer an excellent opportunity for bringing the results *and the method* of physics to many persons who would otherwise have no contact with them.

One step toward this goal would be amendment of the bill to require that the state boards for vocational education be enlarged to include engineers and representatives of the physical sciences; the latter could easily be drawn from the faculties of state universities and land grant colleges. Only with such representation can the challenge offered by the wide extension of vocational-technical training proposed by the bill be met successfully.—J. D. E.

RECENT MEETINGS

Indiana Chapter

The Indiana Chapter of the American Association of Physics Teachers met at Wabash College, Crawfordsville, on November 4, 1944. Professor M. E. Hufford presided at the afternoon session, at which the following program was presented.

Suggestions for a fundamental approach to the treatment of units in elementary textbooks. D. Roller, *Wabash College.*

Physics and good English. J. D. Elder, *Wabash College.*

Some unique features of the new physics building at Oberlin College. L. W. Taylor, *Oberlin College.*

Geometric methods for calculating moments of inertia. I. Walerstein, *Purdue University.*

(*a*) **CAA and FCC type tests;** (*b*) **a vocational guidance pamphlet in physics.** J. B. Hershman, *Valparaiso Technical Institute.*

General physics for engineers—a postwar planning program. I. Walerstein, *Purdue University.*

On the teaching of the basic sciences in a proposed federally subsidized program. K. Lark-Horovitz, *Purdue University.*

A device to show constancy of angular momentum in rotation. M. E. Hufford, *Indiana University.*

An integrated science program in the schools. R. W. Lefler, *Purdue University.*

At a short business session it was voted to continue the committee on teacher training requirements for Indiana schools, of which K. Lark-Horovitz was chairman. At Professor Lark-Horovitz' request, he was replaced by R. W. Lefler. It was voted to accept the invitation of the Purdue University department to hold the next meeting at Lafayette; Professor Lark-Horovitz becomes chairman for that meeting. After the business session tea was served to members, wives and other guests.

The afternoon session was attended by 45 persons, of whom the following are members of the Association:

Brother Bruno, *Cathedral High School, Indianapolis;* E. Ritchie and O. H. Smith, *De Pauw University;* G. D. Van Dyke, *Earlham College;* W. H. Billhartz, *Franklin College;* R. E. Martin, *Hanover College;* A. L. Foley, C. Hire, M. E. Hufford, R. R. Ramsey and W. M. Schwarz, *Indiana University;* L. W. Taylor, *Oberlin College;* K. Lark-Horovitz, R. W. Lefler, I. Walerstein and R. B. Withrow, *Purdue University;* J. W. Alinsky and J. B. Hershman, *Valparaiso Technical Institute;* H. W. Moody, A. R. Thomas and D. Thurman, *Valparaiso University;* W. E. Bixby, J. D. Elder, S. Wray and D. Roller, *Wabash College.*

A dinner at the Crawfordsville Country Club was attended by 64 persons. Professor O. H. Smith was toastmaster for the evening session. After remarks of welcome by President Frank H. Sparks of Wabash College, the meeting was addressed by Professor L. W. Taylor, President of the American Association of Physics Teachers, on "The central place of physics in any liberal arts program."

Fall Meeting of the New England Section

The twenty-fourth regular meeting of the New England Section of the American Physical Society was held at Brown University, Providence, Rhode Island, on October 28, 1944. At least 60 members were in attendance.

Five invited papers were presented:

The British Society for Freedom in Science. P. W. Bridgman, *Harvard University.*

The applied mathematics program at Brown University. R. G. D. Richardson, *Brown University.*

The electrical properties of polarized metal-solution interfaces. David C. Grahame, *Amherst College.*

Some metallurgical applications of radioactive tracers. John W. Irvine, Jr., *Massachusetts Institute of Technology.*

Continuity in mathematics and physics. Morton Masius, *Worcester Polytechnic Institute.*

Eleven contributed papers were presented, abstracts of four of which follow. Abstracts of the remaining contributed papers will be found in the *Physical Review.*

Conversion of nonrationalized cgs to rationalized mks units in electromagnetism. Herbert Jehle, *Harvard University.* The standard method of converting the numerical values of a physical quantity from one unit to another is the substitution of units whereby a physical quantity is considered as an invariant with respect to change of units; for example, 37 erg $=37\times(10^{-7}$ j$)=37\times10^{-7}$ j. The conversion of the numerical values of **E**, **B**, **H** and nearly all other electromagnetic quantities may be performed by such a substitution. With the usual choice of units, however, this procedure fails for electrostatic flux and for **D**, because the rationalization of Gauss' law was performed by a change of the physical quantity of D by a factor 4π. This of course invalidates the substitution method. The most convenient way to solve this dilemma is to keep consistently to the substitution method for converting units and to choose the units of electrostatic flux, in cgs, nonrationalized mks, and rationalized mks units, respectively, as: 1 statlorentz $=1$ lorentz$/3\times10^9=1$ coulomb$/4\pi\times3\times10^9$. This proposed introduction of a new cgs unit, the *statlorentz*, does not involve any change of numerical values and therefore leaves all physical equations untouched, as it is the numerical values only that enter into the equations.

A classical experiment illustrating the notion of "jerk." P. Le Corbeiller, *Harvard University.* The following experiment is described in many textbooks. A heavy weight hangs from a thread, and another piece of thread hangs below the weight. A slow pull on the lower thread breaks the upper thread; a quick downward snap breaks the lower one. To obtain this result, it is necessary that the applied force increase with time. If the rate of change is small, the upper thread breaks; if it exceeds a certain limiting value, the lower thread breaks. Thus the explanation is not as elementary as is usually assumed. The experiment

is rather an example of those phenomena that depend upon the rate of change of acceleration (or "jerk") and that are of importance in the design of automobiles and elevators.

A simple centroider. P. L. Taulbee, *U. S. Coast Guard Academy.* (Introduced by J. B. Hoag.) A description is given of a simple apparatus used in the determination of the center of gravity of irregular objects such as a model ship used in an experiment on metacentric height.

A solenoid experiment. Charles H. Tindal, *Harvard University.* The application of electronic devices to industry has emphasized the use of solenoids as a means of producing controlled forces. Although beginners in physics are told in some detail about solenoids, experiments on them are seldom included in laboratory courses. An interesting experiment is as follows: (1) measure the dimensions (length, inside and outside diameters) of a small solenoid of known number of turns and given wire size, and calculate the maximum thrust to be expected on a given armature; (2) measure the force exerted on the armature as a function of the fractional length inserted, checking maximum force with that calculated in (1); (3) vary the current in the solenoid and plot the thrust as a function of the current; (4) observe the effect of coating the solenoid with iron; (5) assemble the armature in such a manner as to produce a "push-out" rather than a "pull-in" solenoid.

At the business meeting the following officers were elected for the ensuing year: *Chairman,* Morton Masius; *Vice Chairman,* R. B. Lindsay; *Secretary-Treasurer,* Mildred Allen; *Program Committee,* V. E. Eaton and Francis W. Sears.

MILDRED ALLEN,
Secretary-Treasurer

Kentucky Chapter

The Kentucky Association of Physics Teachers met at the University of Kentucky, Lexington, on October 28, 1944. Professor O. T. Koppius of the University of Kentucky presided. The program was as follows:

Magnetic energy and the Zeeman effect. F. W. WARBURTON, *University of Kentucky.*

An adventure in teaching spectroscopy. R. A. LORING, *University of Louisville.*

High voltage generators. C. B. CRAWLEY, *University of Kentucky.*

A demonstration of color illumination. T. L. YOUNG, *Department of Art, Morehead State Teachers College.*

These units. D. M. BENNETT, *University of Louisville.*

Some lecture demonstrations in elementary physics. W. S. WEBB AND J. SCHROEDER, *University of Kentucky.*

The following officers were elected for the ensuing year: President: W. C. WINELAND, *Morehead State Teachers College;* Vice-president: F. W. WARBURTON, *University of Kentucky;* Secretary-treasurer: C. ADAMS, *University of Louisville;* Chapter representative to AAPT: W. NOLL, Berea College.

W. C. WINELAND,
Secretary-Treasurer

Meeting of the Oregon Chapter

The Oregon Chapter of the American Association of Physics Teachers met at Willamette University, Salem, Oregon, on November 4, 1944. Professor W. B. Anderson of Oregon State College presided. The following papers were presented:

Mechanical electrical analogies. F. B. MORGAN, *Oregon State College.*

Rating of physics departments. A. A. KNOWLTON, *Reed College.*

Report on the New York meeting of the Optical Society of America. W. WENIGER, *Oregon State College.*

House conditioning by the reverse cycle. W. R. VARNER, *Oregon State College.*

Demonstration of the megohmmeter. G. VASSALLO, *University of Portland.*

Electrical properties of wood. E. H. COLLINS, *University of Oregon.*

Following a dinner, a talk on "Physics at Naval Proving Grounds" was given by R. R. Dempster, of Oregon State College, who had recently returned from the Naval Proving Ground, Dahlgren, Virginia. He mentioned the many problems of the Bureau of Ordnance and described the scientific battle waged between shell penetration and armor-plate protection.

The next meeting of the chapter will be with the Oregon Academy of Science, probably in January at Portland, and the last meeting of the current academic year will be at Salem on April 28, 1945. Professor W. Weniger, Oregon State College, was elected national representative of the chapter.

E. HOBART COLLINS
Secretary

Physics Club of Philadelphia

The first meeting of the Physics Club of Philadelphia in 1944–45 was held in the Randal Morgan Laboratory of the University of Pennsylvania on October 20. An address on "Analytical mechanics in dentistry" was given by Dr. Arthur B. Gabel of the School of Dentistry, University of Pennsylvania. The speaker pointed out that whereas the "commonsense" approach to mechanical problems in dentistry has frequently failed, some of these problems have been solved by the application of the principles of analytical mechanics.

At the second meeting of the year, held jointly with the Franklin Institute on December 6, Dr. G. C. Southworth, of Bell Telephone Laboratories, spoke on "New experimental technics for use with centimeter waves." The lecturer traced briefly the changes in technic used in proceeding to higher and higher frequencies, and described in some detail a relatively new method, involving transmission in hollow metal pipes and resonance in cavities, that greatly facilitates the practical use of frequencies above 3000 megacycle/sec.

MABEL A. PURDY,
Secretary-Treasurer

DIGEST OF PERIODICAL LITERATURE

Gas Law Demonstration Apparatus

A 100-ml hypodermic syringe with a glass barrel is provided with a stopcock at the delivery end and is mounted in one end of a copper tube some 30 in. long, in which a long slot has been cut. Three legs are attached to this end of the tube so that it can be stood vertically on the table, with the syringe at the bottom and the top of the piston visible in the slot. By means of the stopcock the volume of enclosed air is adjusted to 100 ml. Cylindrical weights, consisting of lead-filled copper tubes of appropriate lengths, are then successively lowered onto the piston by means of a cord. These weights, when adjusted to change the volume of the air by, say, 10 ml at a time, range from about 3 to 26 lb. From a knowledge of the area and weight of the piston, the atmospheric pressure, the weight applied and the volume as read from the barrel of the syringe, Boyle's law can be verified. The apparatus is better adapted to demonstration than is the more usual form involving mercury columns of variable height, and the application of pressure by the weights is more direct and easily understood by students. The syringe can be surrounded with a glass water jacket for the verification of Charles' law.—F. C. HICKEY, J. Chem. Ed. **21**, 491–2 (1944).

Model to Demonstrate Elastic and Plastic Properties

The barrel of a 2-ml all-glass syringe is held vertically and connected above to the single arm of a two-way stopcock. One of the two arms on the upper side of the stopcock is a capillary tube leading to the bottom of a glass vessel of about the size of the barrel of the syringe. The third stopcock arm is a tube of normal diameter that passes through the center of the bottom of the vessel and reaches nearly to its top. To the top of the vessel is attached a one-way stopcock. The vessel is filled with lubricating oil of medium viscosity.

When the upper stopcock is closed and the lower is turned so that the barrel of the syringe communicates with the tube passing through the vessel, a weight applied to the plunger of the syringe will draw the plunger downward. The pressure of the air in the central tube and above the oil in the vessel is reduced until the plunger comes to equilibrium. When the weight is removed the plunger returns to its former position. These motions are rapid and correspond to the motions of a spring. If the lower stopcock is turned the other way, the oil from the upper vessel can flow through the capillary into the syringe when a weight is applied to the plunger and return to the vessel when the weight is removed. The curve of piston displacement *versus* time is then like that representing the retarded elastic behavior of many high polymers. If the upper stopcock is open, the lower one being connected to the capillary, the motion of the plunger under the action of a force is like that of plastic flow. More intricate models of this kind can be built to illustrate more elaborate systems. —G. GOLDFINGER and C. B. WENDELL, JR., J. Chem. Ed. **21**, 434–5 (1944).

Simple Apparatus in Teaching and Research

In his presidential address before the Essex Science Teachers' Association, Lord Rayleigh discussed the question: "Are expensive appliances necessary?" In the student laboratory much can be learned about the Wheatstone bridge by using a wire stretched along a rough board, graduated with ink or pencil marks, and a piece of metal held in the hand to make contact with it. From a purely teaching point of view this is as good, if not better, that a postoffice box costing as many pounds as the other does pence. It is much less likely to muddle the beginner and probably will give him more insight into the physics involved. If the student has rigged it up for himself he will further get a sense of independence and achievement that he never gets by handling the elaborate product of the instrument maker. Many people regard an optical instrument not as an arrangement of reflecting and refracting surfaces, but as a construction of lacquered brass. The schoolboy first regards the telescope as a thing that "pulls out." All the essentials of the instrument can be better appreciated by sticking lenses with Plasticene on a strip of wood. Helmholtz as a boy made his own telescope out of spectacle lenses and a cardboard tube. In commercial instruments many essential parts are rightly hidden from view by protective devices, and the beginner is deterred from meddling with them for fear of damaging valuable property.

Lord Rayleigh asserted that beginners in research also should be "graduates in the school of string and sealing wax." Faraday is an example; also Maxwell, in such constructions in the Cavendish Laboratory as his model of the thermodynamic surface. Charles Parsons, the greatest mechanical engineer of his generation, could deal with formidable problems of large-scale construction; yet he found paper, sealing wax, wire and steel knitting needles very adequate materials for making a working model of his air turbine. When the late Lord Rayleigh had occasion to set up a pair of mirrors for Fresnel's interference experiment he mounted them in a few minutes on two lumps of soft wax. The amateur method is not only much cheaper but also often takes little more time than the dispatch of an order to the instrument maker. Progress, too, can be made while the mind is red-hot upon the project, before delay has cooled enthusiasm.

Little sympathy is shown for the research worker who puts the responsibility for designing his apparatus upon an instrument firm. Rutherford is quoted as saying that, if necessary, he could carry out research at the North Pole. J. C. McConnel, when compelled for health reasons to winter in Switzerland away from laboratories or facilities of any kind, still made important observations on the crystallization of ice, noting in particular the size of the individual crystals and the behavior under bending forces when single crystal rods were cut in various directions. Lord Rayleigh paid a tribute to the instrument making

industry and its part in practical life and nonpioneering research.—ANON., *Nature* **154**, 392 (1944).

Density Plummets

An accurately adjusted density plummet is an excellent means for indicating changes in the density of a liquid with temperature. The plummet can be made by drawing down a test tube at the middle to a diameter of about 1/16 in., breaking it there, properly weighting the lower portion and sealing the neck in a flame. Water colored with ink is best for weighting. A medicine dropper drawn down fine enough to enter the neck of the plummet is convenient for filling it. A plummet made to float in water at a particular temperature will sink if the temperature rises a few degrees. The critical temperature may be marked on the plummet with a grease pencil. It is possible, though somewhat difficult, to adjust such a plummet to float in water at 4°C but sink when the temperature rises or falls.—C. E. LLOYD, Sch. Sci. and Math. **44**, 785–8 (1944).

Physics and Surgery

Force and movement.—Two most important topics in physics are force and movement, and of these we have endless examples in the human body. The movement, of course, occurs at the joints, and the force is exerted by the muscles; but few of us appreciate the exquisite precision of the mechanism by which the movements are controlled or the forces that the muscles can exert. The best examples are always obtained from one's own experience, and I cannot choose a better one than an exercise of which in years gone by I had ample experience—the pulling of an oar. Seated on a sliding seat and driving with his full power from the stretcher, an oarsman can exert a drive of 400 lbwt. As his seat moves freely, it follows that through each leg he exerts 200 lbwt. Let us see how this is accomplished.

Although apparently simple, the movements and forces involved actually are exceedingly complex. The most important movement is the straightening of the knee, and the muscle effecting this is the quadriceps extensor. Now in my own case, while the slide travels 18 in., my patella travels only 3 in., or one-sixth of the track of the slide. Therefore, if the quadriceps alone did the whole of the work, the force exerted by the quadriceps on the patella would be six times the driving force of the leg, or 1200 lbwt. This is an exaggeration, as other muscles act indirectly, and noticeably the gluteus maximus in extending the hip; but I conservatively estimate that my quadriceps alone can exert 600 lbwt, and that the whole of my thigh muscles acting together can exert half a ton. Moreover, these large forces have to be exerted with the utmost delicacy, for the watermanship that controls the balance of a racing boat depends upon differences in the thrust of the toes measured in ounces.

Now the modern treatment of a fractured femur is to reduce the fracture to an accurate position by traction, and usually a force of 15 lbwt, is quite sufficient. How does one account for such a ridiculous disparity? To talk of tiring out the muscles is absurd, for the effect on the patient's comfort is immediate. What really happens is that the irritation arising from the fracture is checked, the reflex producing spasm of the muscles is broken, the muscles relax, and it is on the relaxed muscles that traction is exerted. It is important that this point be remembered when the powerful methods of modern skeletal traction are employed—its neglect has often meant failure of a simple fracture to unite.

If such a small force is to be effective it is essential that the limb should be slung quite freely so as to avoid friction; if this is done the traction force has an interesting secondary result. There must, of course, be some counter-pull to prevent the patient from being pulled out of bed, and this is most easily obtained by raising the foot of the bed. But obviously this counter-pull acts on the center of the pelvis, while the traction pull passes through the hip. This side of the pelvis is thus pulled down so that the pelvis lies obliquely and the leg is effectively abducted at the hip. In the case of a fractured femur this is unimportant, but in many conditions of the hip joint abduction is the essential point in treatment. One might imagine that this would easily be obtained by swinging the leg out; but this is by no means true, as the pelvis may swing with it. Traction in the line of the leg is the correct physical method of obtaining abduction; but this simple fact is not generally appreciated, and the so-called abduction splint is still too often seen, even—low be it spoken—in the wards of orthopedic surgeons. Moreover, traction in the line of the femur can abolish pressure on the hip joint, and in the case of inflammation in the joint this may effect the immediate abolition of reflex spasm and the dramatic relief of the patient. Direct attempts at abduction will increase the pressure and, by increasing spasm, make abduction impossible.

A most interesting muscle from the physical standpoint is the biceps muscle of the arm. As the name implies, it arises by two heads from the scapula; it lies in front of the humerus, to which it has no attachment, and is inserted into the back of a tubercle on the inner side of the radius. Its action is thus a very complex one. It rotates the forearm outwards, it flexes the elbow, and it flexes the shoulder; and thus it achieves the most important of all human movements—it enables us to bring food to our mouths.

For the moment consider only the rotation of the forearm. The very powerful movement of supination is effected by two muscles, the biceps and the supinator brevis, both acting at great mechanical disadvantage and the biceps taking the larger share. By holding a weight at the end of a rod and raising it by rotating the forearm it is easy to measure the force involved and to show that the biceps tendon, which is no bigger than a shoelace, can easily support a tensile force of at least 100 lbwt.

But the biceps does far more than merely provide a large force: by its nervous connections it measures the force and adjusts it to requirements, and at the same time it acts as a delicate feeling piece, defining exactly the degree of its contraction and giving most precise information as to the position of the limb. How superbly the muscles perform this function can be seen in the facility with which

a pianist can strike a distant key with absolute certainty and without aid of vision.

The position of the hand and arm produced by contraction of the biceps, supination, and flexion of the elbow and shoulder is of particular interest. This is the position adopted by the violinist and the fencer, and, far from being abnormal, is the position in which the fingers and the whole limb are under the most perfect control. Probably it has never occurred to a violinist that it is to his biceps that he owes the possibility of maintaining the position of his left hand. Still less would it occur to him that the peculiar action of the *right* hand in drawing the bow, described as raising the wrist, is essentially based on a rotation of the forearm, again controlled by the biceps. Rotation of the forearm is indeed a movement of countless uses, and one over which it is possible to develop the most exquisite control.

Another muscle with a fascinating mechanism is the opponens of the thumb. This little muscle can exert a large force, and with the adductor pollicis forms the basis of a powerful grip. But the beauty of its action consists in the *rotation* which it gives to the thumb so that this may face the object grasped. Whatever the size of the object—large ball, ring, pencil—it will be found that the pulp of the thumb faces the object and insures a powerful and yet most delicate grasp. It is one of the best examples of adaptation to function in the human body.

Force and the skull.—So far we have considered the forces exerted *by* the body. Consider now the forces it is capable of resisting, for example, the effects of a blow on the head. A man struck on the head by a 1-lb stone which had fallen 9 ft would probably have some degree of concussion, although his skull might not be broken. If the falling stone deformed the skull ¼ in., which is about the possible limit, the average stopping force would be 432 lbwt. If this were distributed over an area of the skull 2½ in. in diameter, the local pressure would be 86 lb/in.^{2}.

Now, the skull is for all practical purposes filled by an incompressible fluid. Hence any sudden deformation will be resisted by a rise in intracranial pressure, the effect of which will be enormously greater than any resistance offered by the comparatively elastic skull. Thus the rise in intracranial pressure in the case of the stone will be at least 80 lb/in.^{2}. Such a pressure, even though only momentary, must force every drop of blood from the capillaries and leave the brain completely anemic, with the clinical result of a complete and instantaneous paralysis. Surely this provides complete justification for Trotter's theory that the phenomena of concussion are the result of sudden anemia resulting from the rise in intracranial pressure.

My attention was first directed, many years ago, to the problem of intracranial pressure by a curious case at the London Hospital. A child of 3 years had been knocked down by a light cart, the wheel of which passed over her head. She was unconscious and had a large, dirty lacerated wound of the scalp above and behind the right ear. While trying to clean the wound, I noticed a fine crack running vertically through the center of the temporal region. The child's condition precluded any further disturbance, and indeed I was astonished two days later to find her fully conscious and in very fair condition. Then, however, she went back, and in another two days was comatose, with clear signs of intracranial pressure. Expecting to find an intracranial hemorrhage, I trephined and discovered beneath the hairlike crack not blood but an extradural disk of hard mud, 2 in. in diameter and ½ in. thick in the center. I removed the mud and left the skull widely open. To my amazement the child made an uninterrupted recovery.

Now, the considerable mass of mud can only have got in through the crack, and as the force applied was but momentary, the crack must have opened widely. If we estimate the momentary force as 250 lbwt, this would be supported only by the pressure within the skull, for a child's skull is far too fragile to offer appreciable resistance to such violence. If, further, we regard the child's skull as a 5-in. sphere and imagine it to have been flattened by the wheel over an area 2½ in. in diameter—a reasonable approximation—we find that the 250-lb force must have been impressed on an area of 5 in.^{2}, with a resulting pressure within the skull of 50 lb/in.^{2}. Now the central cross-sectional area of this sphere is 20 in.^{2}, and as the internal pressure, although momentary, is exerted equally in all directions, this means a disruptive force of nearly half a ton, which fully explains the wide opening of the crack. That such forces must produce anemia is obvious; that the brain should recover shows how little even the most delicate cells are affected by mere pressure and that always its most important effect is to deprive them of their blood supply.

Absorption of heat.—An interesting problem in physiological physics, and one with practical results of the first importance, is concerned with the application of heat to the surface of the body. As a simple example, consider what happens if the forearm is immersed in water at 110°F. If the forearm were a mere mass of inert flesh its temperature would rapidly rise to that of the water; but that this does not occur is shown by a very simple experiment due to Sir Thomas Lewis. If the circulation of the immersed arm is checked, in a few seconds the feeling of comfortable warmth gives place to intolerable pain, for the deep layers of the skin are too sensitive to stand such a temperature. With an intact circulation the skin vessels rapidly dilate and the heat is carried off by the blood stream, so that the temperature within the limb does not rise to a painful level. The temperature in the deep tissues does, however, rise somewhat, and in response to this, the vessels dilate and the blood flow increases. This necessarily results in increased metabolism and an internal production of heat in the limb.

An elaborate protective mechanism is thus brought into play, depending for its effectiveness upon an intact circulation, and in this the skin plays a predominant part. The skin protects the deep tissues in two ways: when cold and avascular it forms an efficient defense against loss of heat, without which one could survive in a cold sea only for a short time; when warm its vascularity increases enormously, so that the blood flow may be a hundred times as great, and it now protects the deep structures because the heat is carried away by the blood.

If, however, owing to local pressure or some defect in circulation the blood flow in the skin is diminished, this protective mechanism breaks down and the temperature of the skin itself, or even of the deeper structures, may rise until coagulation and local death of the tissues occur. Doubtless this is the explanation of many hot-water-bottle burns, especially under anesthesia and in patients with defective circulation.

Endothermy is one of the most valuable gifts of physics to surgery, and with reasonable precautions and in skilled hands would seem to be perfectly safe. It consists in the passage through the body of a high voltage current of frequency of the order of 10^6 sec^{-1}—so high, in fact, that none of the chemical effects of electricity occur in the tissues. Since the current is comparatively large, as much as 3 amp, considerable heat is produced by the resistance of the tissues; and as the name endothermy implies, this heat is produced *in* the tissues. If, for example, the two poles of the apparatus are held in the hands, the only sensation is one of peculiar warmth inside the wrists and spreading up the arms.

Now, provided that the heat is carried off by the circulation so that the local rise of temperature is not excessive, no harm results. If, however, this condition is not satisfied, terrible disasters may occur. It is quite common to place the so-called indifferent pole under the sacrum, and, unless a large soft pad is used, the pressure on the thin skin in this region may render the skin avascular, so that its temperature rises to a dangerous point. But this is not all. The sacrum itself has a poor blood supply, and is not very sensitive, so that even in a conscious patient its temperature may rise dangerously. I know of two cases in which it underwent total necrosis. The risk of such a disaster is very real, and the indifferent endothermy pole should never be placed in this position. In fact, wherever it is applied care must be taken that the blood supply of the tissues is adequate and not obliterated by pressure; otherwise, extensive necrosis of the skin may occur.

The increased metabolism arising from the heating of tissues introduces considerations which show how dangerous it is to regard the body as a mere physical apparatus, and this is especially the case where there is a defect in the circulation. If heat is applied to a limb in which the circulation is so defective that it cannot respond to the demands made upon it, gangrene may be precipitated. If, on the other hand, heat is applied to some other part of the body no harm can be done, since heat can be conveyed to the damaged limb solely by its circulation, and increase in this can only be beneficial.

Similar considerations apply in certain cases of shock; for if the body is heated when the amount of circulating blood is inadequate, the resulting vasodilatation and increased metabolism may actually result in a further lowering of the blood pressure, with even fatal results. Supplying heat is justified only when one is sure of adequate blood in active circulation.

Radiation.—Gamma-rays profoundly affect the tissues, and especially cellular growth. Cells that are dividing rapidly may be completely destroyed, and it is this property that gives these rays their peculiar value in the treatment of malignant growths. In such growths the cells are always dividing at a rate many times, sometimes thousands of times, more rapid than that of normal cells, and it is just this rapid division which marks them as vulnerable to x-rays and gamma-rays.

But the normal tissues are also profoundly affected, and the greatest skill is needed to destroy a growth by radiation without doing irreparable damage to the surrounding tissues. Moreover, if the growth is not completely destroyed and reappears, it will now spread in the damaged tissues, with annihilating effect. Some sacrifice of normal tissue is unavoidable, but only to such extent as can in time be remedied by the natural process of repair.

Of the exact mechanism by which the radiation produces its effect upon the cell we know almost nothing. Certainly it must be atomic, and primarily an ionization of certain atoms and consequent rise in their chemical activity. Some substance is thus produced that is toxic to the cell, or perhaps some substance is destroyed that is essential to its normal growth. Apparently it is upon the chromosomes and the reproductive mechanism of the cell that the effect is most severe, for we know that the cells which survive and continue to reproduce are themselves deformed and transmit their defects to succeeding generations. Radiation evidently is one of the greatest weapons of surgical progress; but it is indeed a weapon, not a tool, and only those who are prepared to devote their lives to its study have any right to use it except in very limited fields or with the close coöperation of those who understand its dangers as well as its powers.

Conclusion.—I have ranged over a wide field and yet have touched only the fringes of my subject. I hope to have convinced you that at many points the work of the surgeon enters the field of physics and that towards the solution of many surgical problems the physicist could give invaluable help. The future should see a fuller collaboration between two branches of science that at first seem to have little in common.—H. S. Souttar, 1943 Bradshaw Lecture before the Royal College of Surgeons, British Med. J. **2**, 737 (1943).

Check List

Report on the teaching of mathematics to physicists. Anon., *J. Sci. Inst.* **21**, 127 (1944). This note briefly describes a report prepared by a joint committee of the British Mathematical Association and the Institute of Physics. For a copy of the report, address the Secretary, Institute of Physics, The University, Reading, Berks., Eng.

Recent progress in scientific computing. L. J. Comrie, *J. Sci. Inst.* **21**, 129–135 (1944). Modern calculating machines and mathematical tables and their influence on our attitude toward computing.

The planning of a glassworking department. R. L. Breadner and C. H. Simms, *J. Sci. Inst.* **21**, 169–173 (1944). The room layout, machines, burners, oven and supplies.

Harmonic synthesizer for demonstrating and studying complex wave forms. J. M. Somerville, *J. Sci. Inst.* **21**, 174–177 (1944). This device will combine any seven sinoidal

electric oscillations in the range 50 to 2×10^4 cycle/sec and allow separate control of the amplitude and phase of each component. It may be used to study many types of combined oscillations—especially the combination of a fundamental and several harmonics to form a nonsinoidal oscillation, and the effect of the relative amplitudes and phases of the components on the resulting wave-form. It may also be used for harmonic analysis of a given oscillation.

A scientific investigation of a fake cosmic-ray "concentrator" or "philosopher's stone." R. H. O., *J. Franklin Inst.* **238**, 136 (1944). According to G. L. Clark [*Industrial Radiography* (vol. 2, No. 3)], there have recently been sold in Illinois about 70 devices at $300 each for which was claimed cure of every type of disease. The sales campaign included "a lecture in which the longest possible words descriptive of nuclear physics and cosmic rays, and invariably mispronounced, were strung together . . . with the result that each sentence was completely meaningless from the standpoint of scientific validity." The device was supposed to absorb cosmic rays and re-emit them in some highly beneficial form into the pathological lesion or organ over which it was placed. The manufacturing cost apparently was about 50 cts.

A bibliography of electron microscopy. C. Marton and S. Sass, *J. App. Phys.* **15**, 575–579 (1944). An addendum to the earlier, comprehensive list [*J. App. Phys.* **14**, 522 (1943)].

Etymology of the word microradiograph. S. E. Maddigan, *J. App. Phys.* **15**, 626 (1944).

Physics in the service of Southern agriculture. Staff of Southern Research Laboratory, *J. App. Phys.* **15**, 629–641 (1944).

Scientific and industrial research. Anon., *Nature* **154**, 283, 311, 346, 373 (1944). This series of comprehensive editorials also discusses the relations of universities to research.

Current problems of visual research. W. S. Stiles, *Nature* **154**, 290–293 (1944). This discussion mainly concerns the sensitivity of the retina.

Problems of modern physics. J. Frenkel, *Nature* **154**, 450–454 (1944). A brief survey of atomic nuclei, elementary particles, and the nature of matter.

Foundations of electrical measurements. L. Hartshorn, *Nature* **154**, 534–537 (1944). Suggests some reasons for the great divergence of opinion expressed in recent articles on the precise significance of electric and magnetic units.

The reception of Ohm's electrical researches by his contemporaries. H. J. J. Winter, *Phil. Mag.* **35**, 371–386 (1944).

Experiments on the six focal lines due to reflections at the surfaces of two plano-cylindrical lenses. G. F. C. Searle, *Phil. Mag.* (7) **35**, 477–491 (1944). Experiments for students.

The electromagnetic spectrum as an analytical tool. D. M. Gans, J. Chem. Ed. **21**, 421–9 (1944). A survey of the instruments used in and the data obtained from infrared, visible and ultraviolet spectroscopy, absorption and reflectance spectrophotometry, visible and electron microscopy and x-ray diffraction.

Bequest to the Academic Youth of His Country

What shall I wish for the young students of my country? First of all, sequence, consequence and again consequence. In gaining knowledge you must accustom yourself to the strictest sequence. You must be familiar with the very groundwork of science before you try to climb the heights. Never start on the "next" before you have mastered the "previous." Do not try to conceal the shortcomings of your knowledge by guesses and hypotheses. Accustom yourself to the roughest and simplest scientific tools. Perfect as the wing of a bird may be, it will never enable the bird to fly if unsupported by the air. Facts are the air of science. Without them the man of science can never rise. Without them your theories are vain surmises. But while you are studying, observing, experimenting, do not remain content with the surface of things. Do not become a mere recorder of facts, but try to penetrate the mystery of their origin. Seek obstinately for the laws that govern them. And then—modesty. Never think you know it all. Though others may flatter you, retain the courage to say, "I am ignorant." Never be proud. And lastly, science must be your passion. Remember that science claims a man's whole life. Had he two lives they would not suffice. Science demands an undivided allegiance from its followers. In your work and in your research there must always be passion.—Ivan Pavlov.